OSP Journal of Physics and Astronomy

Research Article

On Gravity and Nuclear Force Unification and Gravitomagnetic and Colormagnetic Higgs Bosons and Photons

Delso J

Bachelor's Degree in Physics, Zaragoza University, Spain

Corresponding Author: Jesus Delso Lapuerta, Bachelor's Degree in Physics by Zaragoza University, Spain.
E-mail: jesus.delso@gmail.com

Received: November 08, 2023; **Accepted:** November 22, 2023; **Published:** December 02, 2023

Abstract

A new SU(2) X U(1) broken symmetry leads us to gravity and nuclear force unification, new Higgs bosons, intermediate vector bosons and photons appear. Schrödinger's equation for a gravitational system leads us to Planck's gravitational constant and Kepler's third law. The solution of Schröodinger's equation for an electron orbiting a nucleus leads us to the emission of gravitomagnetic photons when the gravitational potential is also taken into account. In four dimensions, the Minkowski metric $\eta_{\mu\nu}$= diag (+1,–1,–1,–1) leads to the 16- dimensional Clifford algebra C(1,3), Dirac's equation [1] is using four of these 16 matrices that form a basis of this algebra, a new operator is defined using all of these matrices and also generalized for a curved space. This new multilevel operator generalizes the Dirac's equation, the value of the generalized Dirac's operator is calculated in the Schwarzschild's metric. The torsion tensor is calculated taking into account the non-symmetric part of the metric tensor in the vanishing of its covariant derivative and applied to Kerr's metric generalizing the Clifford algebra. Geodesic equation, conservation laws, torsion tensor and Einstein field equation are obtained in a non-symmetric geometry.

Keywords: *Broken Symmetry; Intermediate Vector Bosons; Gravity and Nuclear Force Unification; Gravitomagnetic Higgs Boson; Colormagnetic Higgs Boson; Gravitomagnetic Photon; Colormagnetic Photon; Planck's Gravitational Constant; Planck's Constant; Kepler's Third Law; Schrödinger's Equation; Gravitomagnetic Photon Emission; Gravitomagnetic Tensor; Gravitational Magnetic Field; Energy-Momentum 1-Form; Clifford Algebra; Dirac Equation; Dirac Operator; Gravity and Quantum Mechanics Unification; Multilevel Operator; Schwarzschild's Metric; Torsion Tensor; Rearranged Kerr's Metric; Generalized Dirac Equation; Generalized Clifford Algebra; Generalized Einstein Field Equation; Generalized Geodesic Equation; Conservation Laws; Non-Symmetric Geometry*

Introduction

In the 1960s, Sheldon Glashow, Abdus Salam and Steven Weinberg unified the electromagnetic force and the weak interaction by showing them to be two aspects of a single force, now termed the electroweak force. The Higgs mechanism is essential to explain the generation mechanism of the property "mass" for gauge bosons.

Dirac's equation is the relativistic wave equation derived by physicist Paul Dirac in 1928. The wave functions in the Dirac theory are vectors of four complex components (known as bispinors), two of which resemble the Pauli wavefunction in the non-relativistic limit, in contrast to the Schrödinger equation which described wave functions of only one complex component.

Dirac's operator is just the tip of the iceberg, the tip of a generalized operator that is obtained by operating on all members of the Clifford algebra basis and not just on four of them.

The Schwarzschild's metric is named in honour of Karl Schwarzschild, who found the exact solution in 1915 and published it in January 1916, a little more than a month after the publication of Einstein's theory of general relativity. It was the first exact solution of the Einstein field equations other than the trivial flat space solution. Schwarzschild died shortly after his paper was published, as a result of a disease he developed while serving in the German army during World War I. Johannes Droste in 1916 independently produced the same solution as Schwarzschild.

Schwarzschild's metric is an exact solution to the Einstein's field equations that describes the gravitational field outside a spherical mass, on the assumption that the electric charge of the mass, angular momentum of the mass, and universal cosmological constant are all zero.

The new generalized Dirac's operator, the multilevel operator, is calculated in the Schwarzschild's metric, torsion tensor and new gravitomagnetic tensor appear in level 2, curvature tensor appears in levels 3 and 4.

The Kerr's metric is a generalization to a rotating body of the Schwarzschild's metric. The Einstein field equation relates the geometry of spacetime to the distribution of matter within it. The equations were published by Einstein in 1915 in the form of a tensor equation which related the local spacetime curvature with the local energy, momentum and stress within that spacetime expressed by the stress-energy tensor.

Gravity and nuclear force unification. The rainbow unification

In Glashow, Weinberg and Salam model $SU(2) \times U(1)$ is the broken symmetry[2][3] for electromagnetic and weak force unification with weak charges, orange and purple, (o) and (p), and anti charges, anti orange and anti purple, $(\bar{o})$ and $(\bar{p})$. The massless electromagnetic photon γ_e^0 and the weak bosons $W^- W^+$ and Z^0 carry the force between the first generation quarks d and u, and the first generation leptons e^- and ν_e; these bosons are related to Higgs boson H^0 via Higgs mechanism and Planck's constant h has been determined for hydrogen atom system, weak bosons carry the electromagnetic short distance force.

$$\gamma_e^0(m = 0, q_e = 0) \quad (1)$$

$$W^-(m = 80.377\text{GeV}, q_e = -1, q_w = (o\bar{p})) \quad (2)$$

$$W^+(m = 80.377\text{GeV}, q_e = +1, q_w = (p\bar{o})) \quad (3)$$

$$Z^0\left(m = 91.187\text{GeV}, q_e = 0, q_w = \frac{1}{\sqrt{2}}(o\bar{o} - p\bar{p})\right) \quad (4)$$

$$H_e^0(m = 125\text{GeV}, q_e = 0) \quad (5)$$

$h_e^H = 6.626070150000000549 \cdot 10^{-34}$, from equations (109) and (111) (6)

In rainbow unification $[SU(2) \times U(1)]_{du,yk}$ is the broken symmetry for gravitomagnetic and nuclear force unification with weak charges, yellow and pink, (y) and (k), and anti charges, anti yellow and anti pink, $(\bar{y})$ and $(\bar{k})$. The massless gravitomagnetic photon γ_g^0 and the weak bosons $\rho^-\rho^+$ and ρ^0 carry the force between the first generation quarks d and u, and the first generation quarks d and u; these bosons are related to Higgs boson H_g^0 via Higgs mechanism and Planck's gravitomagnetic constant h_g has been determined for Solar systems and for hydrogen atom system, rho bosons carry the gravitomagnetic short distance force.

$$\gamma_g^0(m = 0, q_e = 0) \quad (7)$$

$$\rho^-(m = 775.11\text{MeV}, q_e = -1, q_w = (y\bar{k})) \quad (8)$$

$$\rho^+(m = 775.11\text{MeV}, q_e = +1, q_w = (k\bar{y})) \quad (9)$$

$$\rho^0\left(m = 775.26\text{MeV}, q_e = 0, q_w = \frac{1}{\sqrt{2}}(y\bar{y} - k\bar{k})\right) \quad (10)$$

$$H_g^0(m > 0, ?\,\text{GeV}, q_e = 0) \quad (11)$$

$h_g^H = 1.390715340705763914900048 \cdot 10^{-53}$, from equations (106) and (108) (12)

$h_g^{SE} = 4.667814277790492524887853 6 \cdot 10^{37}$, from equations (94) and (96) (13)

$h_g^{SJ} = 2.836436278840702454154584284 \cdot 10^{41}$, from equations (101) and (103) (14)

In rainbow unification $[SU(3) \times U(1)]_{du,rgb}$ is the broken symmetry for colormagnetic and hadron force unification with weak charges, red, green and blue, (r), (g) and (b), and anti charges, anti red anti green and anti blue, $(\bar{r})$, $(\bar{g})$ and $(\bar{b})$. The massless gravitomagnetic photon γ_c^0 and the weak bosons $C_1^- C_2^+ C_3^- C_4^+ C_5^- C_6^+ C_7^0$ and C_8^0 carry the force between the first generation quarks d and u, and the first generation quarks d and u; these bosons are related to Higgs boson H_c^0 via Higgs mechanism and Planck's colormagnetic constant h_c has to be determined for Deuterium system and for baryon and meson systems, C bosons carry the colormagnetic short distance force.

$$\gamma_c^0(m = 0, q_e = 0) \quad (15)$$

$$C_1^- = C_{\text{durg}}^-(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (r\bar{g})) \quad (16)$$

$$C_2^+ = C_{udgr}^+(m > 0, ?\,\text{GeV}, q_e = +1, q_w = (g\bar{r})) \quad (17)$$

$$C_3^- = C_{durb}^-(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (r\bar{b})) \quad (18)$$

$$C_4^+ = C_{udbr}^+(m > 0, ?\,GeV, q_e = +1, q_w = (b\bar{r})) \quad (19)$$

$$C_5^- = C_{dugb}^-(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (g\bar{b})) \quad (20)$$

$$C_6^+ = C_{udbg}^+(m > 0, ?\,\text{GeV}, q_e = +1, q_w = (b\bar{g})) \quad (21)$$

$$C_7^0 = C_{dduuI}^0\left(m > 0, ?\,GeV, q_e = 0, q_w = \left(\frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})\right)\right) \quad (22)$$

$$C_8^0 = C_{dduuY}^0\left(m > 0, ?\,GeV, q_e = 0, q_w = \left(\frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})\right)\right) \quad (23)$$

$$H_c^0(m > 0, ?\,\text{GeV}, q_e = 0) \quad (24)$$

$h_c^D = ?$, for Deuterium system (25)

$h_c^b = ?$, for baryon system (26)

$h_c^m = ?$, for meson system (27)

In rainbow unification $[SU(2) \times U(1)]_{sc,op}$ is the broken symmetry for electromagnetic and weak force unification with weak charges, orange and purple, (o) and (p), and anti charges, anti orange and anti purple, $(\bar{o})$ and $(\bar{p})$. The massless electromagnetic photon $\gamma_{e,sc}^0$ and the weak bosons $W_{sc}^- W_{sc}^+$and Z_{sc}^0 carry the force between the second generation quarks s and c, and the second generation leptons $\mu^- = e_{sc}^-$and $\nu_\mu = \nu_{sc}$; these bosons are related to Higgs boson $H_{e,sc}^0$ via Higgs mechanism and Planck's constant $h_{e,sc}$ has to be determined for baryon and meson systems, weak bosons carry the electromagnetic short distance force.

$$\gamma_{e,sc}^0(m = 0, q_e = 0) \quad (28)$$

$$W_{sc}^-(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (o\bar{p})) \quad (29)$$

$$W_{sc}^+(m > 0, ?\,GeV, q_e = +1, q_w = (p\bar{o})) \quad (30)$$

$$Z_{sc}^0\left(m > 0, ?\,\text{GeV}, q_e = 0, q_w = \frac{1}{\sqrt{2}}(o\bar{o} - p\bar{p})\right) \quad (31)$$

$$H_{e,sc}^0(m > 0, ?\,\text{GeV}, q_e = 0) \quad (32)$$

$h_{e,sc}^b = ?$, for baryon system (33)

$h_{e,sc}^m = ?$, for meson system (34)

In rainbow unification $[SU(2) \times U(1)]_{sc,yk}$ is the broken symmetry for gravitomagnetic and nuclear force unification with weak charges, yellow and pink, (y) and (K), and anti charges, anti yellow and anti pink, $(\bar{y})$ and $(\bar{k})$. The massless gravitomagnetic photon $\gamma_{g,sc}^0$ and the weak bosons $\rho_{sc}^- \rho_{sc}^+$and ρ_{sc}^0 carry the force between the second generation quarks s and c, and the second generation quarks s and c; these bosons are related to Higgs boson $H_{g,sc}^0$ via Higgs mechanism and Planck's gravitomagnetic constant $h_{g,sc}$ has to be determined for baryon and meson systems, ρ_{sc} bosons carry the gravitomagnetic short distance force.

$$\gamma_{g,sc}^0(m = 0, q_e = 0) \quad (35)$$

$$\rho_{sc}^-(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (y\bar{k})) \quad (36)$$

$$\rho_{sc}^+(m > 0, ?\,\text{GeV}, q_e = +1, q_w = (k\bar{y})) \quad (37)$$

$$\rho_{sc}^0\left(m > 0, ?\,GeV, q_e = 0, q_w = \frac{1}{\sqrt{2}}(y\bar{y} - k\bar{k})\right) \quad (38)$$

$$H_{g,sc}^0(m > 0, ?\,\text{GeV}, q_e = 0) \quad (39)$$

$h_{g,sc}^b = ?$, for baryon system (40)

$h_{g,sc}^m = ?$, for meson system (41)

In rainbow unification $[SU(3) \times U(1)]_{sc,rab}$ is the broken symmetry for colormagnetic and hadron force unification with weak charges, red, green and blue, (r), (g) and (b), and anti charges, anti red anti green and anti blue, $(\bar{r})$, $(\bar{g})$ and $(\bar{b})$. The massless gravitomagnetic photon $\gamma_{c,sc}^0$ and the weak bosons $C_{11}^- C_{12}^+ C_{13}^- C_{14}^+ C_{15}^- C_{16}^+ C_{17}^0$ and C_{18}^0 carry the force between the second generation quarks s and c, and the second generation quarks s and c; these bosons are related to Higgs boson $H_{c,sc}^0$ via Higgs mechanism and Planck's colormagnetic constant $h_{c,sc}$ has to be determined for baryon and meson systems, $C_{c,sc}$ bosons carry the colormagnetic short distance force.

$$\gamma^0_{c,sc}(m = 0, q_e = 0) \quad (42)$$

$$C^-_{11} = C^-_{scrg}(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (r\bar{g})) \quad (43)$$

$$C^+_{12} = C^+_{csgr}(m > 0, ?\,\text{GeV}, q_e = +1, q_w = (g\bar{r})) \quad (44)$$

$$C^-_{13} = C^-_{scrb}(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (r\bar{b})) \quad (45)$$

$$C^+_{14} = C^+_{csbr}(m > 0, ?\,\text{GeV}, q_e = +1, q_w = (b\bar{r})) \quad (46)$$

$$C^-_{15} = C^-_{scgb}(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (g\bar{b})) \quad (47)$$

$$C^+_{16} = C^+_{csbg}(m > 0, ?\,\text{GeV}, q_e = +1, q_w = (b\bar{g})) \quad (48)$$

$$C^0_{17} = C^0_{ssccI}\left(m > 0, ?\,GeV, q_e = 0, q_w = \left(\frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})\right)\right) \quad (49)$$

$$C^0_{18} = C^0_{ssccY}\left(m > 0, ?\,GeV, q_e = 0, q_w = \left(\frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})\right)\right) \quad (50)$$

$$H^0_{c,sc}(m > 0, ?\,GeV, q_e = 0) \quad (51)$$

$$h^b_{c,sc} = ?, \text{ for baryon system} \quad (52)$$

$$h^m_{c,sc} = ?, \text{ for meson system} \quad (53)$$

In rainbow unification $[SU(2) \times U(1)]_{bt,op}$ is the broken symmetry for electromagnetic and weak force unification with weak charges, orange and purple, (o) and (p), and anti charges, anti orange and anti purple, $(\bar{o})$ and $(\bar{p})$. The massless electromagnetic photon $\gamma^0_{e,bt}$ and the weak bosons $W^-_{bt} W^+_{bt}$ and Z^0_{bt} carry the force between the third generation quarks b and t, and the third generation leptons $\tau^- = e^-_{bt}$ and $\nu_\tau = \nu_{bt}$; these bosons are related to Higgs boson $H^0_{e,bt}$ via Higgs mechanism and Planck's constant $h_{e,bt}$ has to be determined for baryon and meson systems, weak bosons carry the electromagnetic short distance force.

$$\gamma^0_{e,bt}(m = 0, q_e = 0) \quad (54)$$

$$W^-_{bt}(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (o\bar{p})) \quad (55)$$

$$W^+_{bt}(m > 0, ?\,GeV, q_e = +1, q_w = (p\bar{o})) \quad (56)$$

$$Z^0_{bt}\left(m > 0, ?\,\text{GeV}, q_e = 0, q_w = \frac{1}{\sqrt{2}}(o\bar{o} - p\bar{p})\right) \quad (57)$$

$$H^0_{e,bt}(m > 0, ?\,\text{GeV}, q_e = 0) \quad (58)$$

$$h^b_{e,bt} = ?, \text{ for baryon system} \quad (59)$$

$$h^m_{e,bt} = ?, \text{ for meson system} \quad (60)$$

In rainbow unification $[SU(2) \times U(1)]_{bt,yk}$ is the broken symmetry for gravitomagnetic and nuclear force unification with weak charges, yellow and pink, (y) and (k), and anti charges, anti yellow and anti pink, $(\bar{y})$ and $(\bar{k})$. The massless gravitomagnetic photon $\gamma^0_{g,bt}$ and the weak bosons $\rho^-_{bt}\rho^+_{bt}$ and ρ^0_{bt} carry the force between the third generation quarks b and t, and the third generation quarks b and t; these bosons are related to Higgs boson $H^0_{g,bt}$ via Higgs mechanism and Planck's gravitomagnetic constant $h_{g,bt}$ has to be determined for baryon and meson systems, ρ_{bt} bosons carry the gravitomagnetic short distance force.

$$\gamma^0_{g,bt}(m = 0, q_e = 0) \quad (61)$$

$$\rho^-_{bt}(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (y\bar{k})) \quad (62)$$

$$\rho^+_{bt}(m > 0, ?\,GeV, q_e = +1, q_w = (k\bar{y})) \quad (63)$$

$$\rho_{bt}^{0}\left(m > 0, ?\,GeV, q_e = 0, q_w = \frac{1}{\sqrt{2}}(y\bar{y} - k\bar{k})\right) \tag{64}$$

$$H_{g,bt}^{0}(m > 0, ?\,\text{GeV}, q_e = 0) \tag{65}$$

$h_{g,bt}^{b} =?$, for baryon system (66)

$h_{g,bt}^{m} = ?$, for meson system (67)

In rainbow unification $[SU(3) \times U(1)]_{bt,rgb}$ is the broken symmetry for colormagnetic and hadron force unification with weak charges, red, green and blue, (r),(g) and (b), and anti charges, anti red anti green and anti blue, $(\bar{r})$, $(\bar{g})$ and $(\bar{b})$. The massless gravitomagnetic photon $\gamma_{c,bt}^{0}$ and the weak bosons C_{21}^{-} $C_{22}^{+} C_{23}^{-} C_{24}^{+} C_{25}^{-} C_{26}^{+} C_{27}^{0}$ and C_{28}^{0} carry the force between the third generation quarks b and t, and the third generation quarks b and t; these bosons are related to Higgs boson $H_{c,bt}^{0}$ via Higgs mechanism and Planck's colormagnetic constant $h_{c,bt}$ has to be determined for baryon and meson systems, $C_{c,bt}$ bosons carry the colormagnetic short distance force.

$$\gamma_{c,bt}^{0}(m = 0, q_e = 0) \tag{68}$$

$$C_{21}^{-} = C_{btrg}^{-}(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (r\bar{g})) \tag{69}$$

$$C_{22}^{+} = C_{tbgr}^{+}(m > 0, ?\,\text{GeV}, q_e = +1, q_w = (g\bar{r})) \tag{70}$$

$$C_{23}^{-} = C_{btrb}^{-}(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (r\bar{b})) \tag{71}$$

$$C_{24}^{+} = C_{tbbr}^{+}(m > 0, ?\,\text{GeV}, q_e = +1, q_w = (b\bar{r})) \tag{72}$$

$$C_{25}^{-} = C_{btgb}^{-}(m > 0, ?\,\text{GeV}, q_e = -1, q_w = (g\bar{b})) \tag{73}$$

$$C_{26}^{+} = C_{tbbg}^{+}(m > 0, ?\,GeV, q_e = +1, q_w = (b\bar{g})) \tag{74}$$

$$C_{27}^{0} = C_{bbttI}^{0}\left(m > 0, ?\,GeV, q_e = 0, q_w = \left(\frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})\right)\right) \tag{75}$$

$$C_{28}^{0} = C_{bbttY}^{0}\left(m > 0, ?\,GeV, q_e = 0, q_w = \left(\frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})\right)\right) \tag{76}$$

$$H_{c,bt}^{0}(m > 0, ?\,\text{GeV}, q_e = 0) \tag{77}$$

$h_{c,bt}^{b} = ?$, for baryon system (78)

$h_{c,bt}^{m} =?$, for meson system (79)

There is no evidence of other types of electrons, for example there is no evidence of e_{dc}^{-}, so we can infer that SU(3) symmetry is not broken and the color force between these quarks is carried by massless gluons.

Planck's gravitational constant

In the hydrogen atom an electron is orbiting a nucleus with 1 proton, we know the energy levels from the solution of the Schrödinger's equation[4], where m_p is the proton mass, m_e is the electron mass, μ is the 2-body reduced mass, e is the electron charge, r is the position of the electron relative to the nucleus, the potential term is due to the Coulomb interaction wherein ϵ_0 is the permittivity of free space.

$$\mu = \frac{m_p m_e}{m_p + m_e} \tag{80}$$

$$V(r) = -\frac{e^2}{4\pi\epsilon_0 r} = -\frac{k_e}{r} \tag{81}$$

$$E_n = -\frac{\mu e^4}{(4\pi\epsilon_0)^2 2\hbar^2 n^2} \tag{82}$$

$$E_n = -\frac{\mu k^2}{2\hbar^2 n^2} \tag{83}$$

with $k = k_e$, now for a gravitational potential

$$V(r) = -\frac{k_g}{r} \tag{84}$$

And in equation (83) we replace $\hbar$ by $\hbar_g$

$$E_n = -\frac{\mu k_g^2}{2\hbar_g^2 n^2} \tag{85}$$

Planck's gravitational constant values

We apply equation (85) to the Sun-Earth system, equating equation (85) to the total energy of the gravitational system we get the value of Planck's gravitational constant in this system, $M_s = 1.9885 \cdot 10^{30}$ kg is the Sun mass, $M_e = 5.97237 \cdot 10^{24}$ kg is the Earth mass, $a = 149598023000$ m is the semi-major axis, eccentricity $e = 0.0167086, k_g = GM_s\mu$, $\mathrm{n} = 1 + 1$ and L the angular momentum. The total energy of the gravitational system is defined by

$$E = -\frac{GM_s\mu}{2a} = -\frac{k_g}{2a} \tag{86}$$

$$\mu = \frac{M_s M_e}{M_s + M_e} \tag{87}$$

$$L^2 = l(l+1)\hbar_g^2 = GM_s a\mu^2(1 - e^2) \tag{88}$$

$$n^2\hbar^2 = (l+1)^2\hbar_g^2 = GM_s a\mu^2 \tag{89}$$

$$\frac{l}{l+1} = \frac{n-1}{n} = 1 - e^2, n = e^{-2} \text{ from } (88)/(89) \tag{90}$$

$$n = 3581.9529381362201856 \tag{91}$$

$$n = 3582, e = 0.0167084902372362 \tag{92}$$

$$E = -\frac{\mu k_g^2}{2\hbar_g^2 n^2} = -\frac{k_g}{2a} \tag{93}$$

$$\hbar_g^2 = \frac{\mu k_g a}{n^2} \tag{94}$$

$$\hbar_g = 7.4290571574528236410479940684343716337342817210645879404396660 45 \cdot 10^{36} \tag{95}$$

$$h_g = 4.66781427779049252488785364223568563336168774518864147659109445351615 \cdot 10^{37} \tag{96}$$

We apply equation (85) to the Sun-Jupiter system, $M_s = 1.9885 \cdot 10^{30}$ kg is the Sun mass, $M_j = 1.8982 \cdot 10^{27}$ kg is the Jupiter mass, $a = 778547261754.2769$ m is the semi-major axis, eccentricity $e = 0.04839266, k_g = GM_s\mu$

$$n = 427.0129152699940647 \tag{97}$$

$$n = 427, e = 0.0483933918495827 \tag{98}$$

$$\mu = \frac{M_s M_j}{M_s + M_j} \tag{99}$$

$$E = -\frac{\mu k_g^2}{2\hbar_g^2 n^2} = -\frac{k_g}{2a} \tag{100}$$

$$\hbar_g^2 = \frac{\mu k_g a}{n^2} \tag{101}$$

$$\hbar_g = 4.51432854542679369402042539318547953563841218096050790100265485011794 \cdot 10^{40} \tag{102}$$

$$h_g = 2.8364362788407024541545842846823831136665704506872014999605746678517 25 \cdot 10^{41} \tag{103}$$

Now we consider the hydrogen atom with $n = 1$, from equations (82) and (83)

$$E = -\frac{\mu k_e^2}{2\hbar^2} = -\frac{k_e}{2a_0} \tag{104}$$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{\mu e^2} = 0.529465409826103847377910923924842418708601351477161933935379748 8 \cdot 10^{-10} \quad (105)$$

Comparing equations (104) and (100) we see the role played by the semi-major axis a now is played by a_0 and from equation (101)

$$\hbar_g^2 = \mu k_g a_0 \quad (106)$$

$$\hbar_g = 2.2133922090705168884053659412952213843762826013967180401998187912 5 \cdot 10^{-54} \quad (107)$$

$$h_g = 1.3907153407057639149004805826049317930723381117551132578366994666 38 \cdot 10^{-53} \quad (108)$$

Now if we adapt equation (106) to k_e we should obtain the Planck's constant value

$$\hbar^2 = \mu k_e a_0 \quad (109)$$

$$\hbar = 1.0545718176461564 \cdot 10^{-34} \quad (110)$$

$$h = 6.6260701500000000549 \cdot 10^{-34} \quad (111)$$

Kepler's third law

If the Hamiltonian is not an explicit function of time, the wave function is separable into a product of spatial and temporal parts [5].

$$\psi(r,t) = \psi(r)e^{\frac{-iEt}{\hbar}} \quad (112)$$

T is the period and Kepler's third law is defined by

$$\frac{GM_s}{a^3} = \frac{4\pi^2}{T^2} \quad (113)$$

E is the total Energy in a gravitational system defined by

$$E = \frac{GM_s\mu}{2a} \quad (114)$$

From equation (89)

$$\hbar = \frac{(GM_s a)^{\frac{1}{2}}\mu}{n} \quad (115)$$

and

$$\frac{E}{\hbar} = \frac{GM_s\mu}{2a}\frac{n}{(GM_s a)^{\frac{1}{2}}\mu} = \frac{n}{2}\frac{(GM_s)^{\frac{1}{2}}}{a^{\frac{3}{2}}} = \frac{n}{2}\omega_m \quad (116)$$

ω_m is the mean motion angular speed defined by

$$\omega_m = \frac{2\pi}{T} \quad (117)$$

$$\omega_m = \frac{(GM_s)^{\frac{1}{2}}}{a^{\frac{3}{2}}} \quad (118)$$

equations (117) and (118) define Kepler's third law and

$$\psi(r,t) = \psi(r)e^{\frac{-in\omega_m t}{2}} \quad (119)$$

Five new planets in Proxima Centauri

Mean motion angular speed ω_m for our planets:

ω_{m1} = 0:0000008266683161721671725893680342060 − *Merucry*

ω_{m2} = 0:0000003236397806290027502923891805337 − *Venus*

ω_{m3} = 0:0000001990958336720942466833404885350 − *Earth*

ω_{m4} = 0:0000001058577386399185014918267545470 − *Mars*

ω_{m5} = 0:00000004324349662 − *Ceres*

ω_{m6} = 0:000000017320508 − *Jupiter*

ω_{m7} = 0:0000000067118273148381163645269302 − *Saturn*

ω_{m8} = 0:0000000023610970045003705167333373453 − *Uranus*

ω_{m9} = 0:0000000012054073971413942728010767548 − *Netpune*

ω_{m10} = 0:0000000008092269920779060844908523775485 − *Pluto* (120)

ω_{m1}/ω_{mn} ratios:

$\omega_{m1}/\omega_{m2} = 2.554285244432914670779337281410185998342$

$\omega_{m1}/\omega_{m3} = 4.152112582796025641132599537390429110737$

$\omega_{m1}/\omega_{m4} = 7.809238387229576415481006972503699305849$

$\omega_{m1}/\omega_{m5} = 19.116592800912295250683362390087871668505$

$\omega_{m1}/\omega_{m6} = 47.727717695818573715584325483178668893545$

$\omega_{m1}/\omega_{m7} = 123.165909579440024344314522761661650849029$

$\omega_{m1}/\omega_{m8} = 350.120437490071555045503244031248304319200979$

$\omega_{m1}/\omega_{m9} = 685.79993629755284246820561633365273445660675$

$\omega_{m1}/\omega_{m10} = 1021.5530676373953341760499255797110784478856049$ (121)

Mean motion angular speed ω_m for planets in Proxima Centauri:

$\omega_{m1} = 0.000014087146623784 -$ Proxima$-d$
$\omega_{m2} = 0.000006513393892588 -$ Proxima $-b$
$\omega_{m3} = 0.0000000398204257868 -$ Proxima $-c$ (122)

ω_{m1}/ω_{mn} ratios:

$\omega_{m1}/\omega_{m2} = 2.162796670383261930601$

$\omega_{m1}/\omega_{m3} = 353.76685069132842946962$ (123)

Comparing equations (121) and (123) we see a gap for 5 planets from ω_{m1}/ω_{m3} to ω_{m1}/ω_{m7}

Gravitomagnetic photon emission

An electron is orbiting a nucleus with Z protons, we know the energy levels from the solution of the Schrödinger equation[4], where m_p is the proton mass, m_e is the electron mass, μ is the 2-body reduced mass, e is the electron charge, r is the position of the electron relative to the nucleus, the potential term is due to the Coulomb interaction wherein ϵ_0 is the permittivity of free space and m_N is the mass of the nucleus.

$$\mu = \frac{m_N m_e}{m_N + m_e} \quad (124)$$

$$V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r} = -\frac{k_e}{r} \quad (125)$$

$$E_n = -\frac{\mu Z^2 e^4}{(4\pi\epsilon_0)^2 2\hbar^2 n^2} \quad (126)$$

$$E_n = -\frac{\mu k^2}{2\hbar^2 n^2} \quad (127)$$

$$C_n = \frac{\mu}{2\hbar^2 n^2} \quad (128)$$

$$E_n = -C_n k^2 \quad (129)$$

with $k = k_e$, now adding the gravitational potential

$$V(r) = -\frac{k_e}{r} - \frac{Gm_N\mu}{r} = -\frac{k_e}{r} - \frac{k_g}{r} = -\frac{(k_e+k_g)}{r} = -\frac{k}{r} \quad (130)$$

And from equation (129)

$$E_n = -C_n\left(k_e^2 + 2k_ek_g + k_g^2\right) \quad (131)$$

$$E_n = -C_n\left(k_e^2 + k_ek_g + k_g^2 + k_gk_e\right) \quad (132)$$

$$E_n^e = -C_n\left(k_e^2 + k_ek_g\right) \quad (133)$$

$$E_n^g = -C_n\left(k_g^2 + k_gk_e\right) \quad (134)$$

$$E_n = E_n^e + E_n^g \quad (135)$$

$$h\nu_n^e = \left(C_n^f - C_n^i\right)\left(k_e^2 + k_ek_g\right) \quad (136)$$

$$h_g\nu_n^g = \left(C_n^f - C_n^i\right)\left(k_g^2 + k_gk_e\right) \quad (137)$$

From equation (137) ν_n^g is the frequency of the gravitomagnetic photon emitted from the initial energy level to the final energy level. This emission leads us to the gravitomagnetic tensor. Gravitational magnetic field generates the extra force needed to explain the anomalous behavior of pendulums observed during a solar eclipse, the Allais effect [7] and also explains the dark matter effect without exotic particles never detected. Gravitational magnetic field is also derived from Special Relativity force transformations [8], when velocities point to the same direction a repulsive gravitational magnetic force is induced. Gravitomagnetic tensor will appear below in equation (88) at level two of the generalized Dirac equation D^2

From equation (136) ν_n^e is the frequency of the electromagnetic photon emitted from the initial energy level to the final energy level. The correction of the second term is an indirect detection of the gravitomagnetic photon emission

$$\alpha = \frac{e^2}{(4\pi\epsilon_0)\hbar c} \quad (138)$$

$$E = -\frac{\mu Z^2 e^4}{(4\pi\epsilon_0)^2 2\hbar^2 n^2}\left[1 + \frac{Z^2\alpha^2}{n}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right)\right] \quad (139)$$

In equation (139) we have the energy levels from the solution of the Dirac's equation [6]. The first term is the solution of the Schrödinger equation that we have seen above in equation (129) and the second term is the relativistic correction

$$E_{nj} = -\frac{\mu Z^2 e^4}{(4\pi\epsilon_0)^2 2\hbar^2 n^2}\frac{Z^2\alpha^2}{n}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right) \quad (140)$$

$$E_{nj} = -\frac{\mu k^4}{2\hbar^4 n^3 c^2}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right) \quad (141)$$

$$C_{nj} = \frac{\mu}{2\hbar^4 n^3 c^2}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right) \quad (142)$$

$$E_{nj} = -C_{nj}k^4 \quad (143)$$

$$E_{nj} = -C_{nj}\left(k_e + k_g\right)^4 \quad (144)$$

$$E_{nj} = -C_{nj}\left(k_e^4 + 4k_e^3k_g + 6k_e^2k_g^2 + 4k_ek_g^3 + k_g^4\right) \quad (145)$$

$$E_{nj} = -C_{nj}\left(k_e^4 + 2k_e^3k_g + 3k_e^2k_g^2 + 2k_ek_g^3 + k_g^4 + 2k_g^3k_e + 3k_g^2k_e^2 + 2k_gk_e^3\right) \quad (146)$$

$$E_{nj}^e = -C_{nj}\left(k_e^4 + 2k_e^3k_g + 3k_e^2k_g^2 + 2k_ek_g^3\right) \quad (147)$$

$$E_{nj}^g = -C_{nj}\left(k_g^4 + 2k_g^3k_e + 3k_g^2k_e^2 + 2k_gk_e^3\right) \quad (148)$$

$$E_{nj} = E_{nj}^e + E_{nj}^g \quad (149)$$

$$h v_{nj}^{e} = \left(C_{nj}^{f} - C_{nj}^{i}\right)\left(k_e^4 + 2k_e^3 k_g + 3k_e^2 k_g^2 + 2k_e k_g^3\right) \quad (150)$$

$$h_g v_{nj}^{g} = \left(C_{nj}^{f} - C_{nj}^{i}\right)\left(k_g^4 + 2k_g^3 k_e + 3k_g^2 k_e^2 + 2k_g k_e^3\right) \quad (151)$$

From equation (150) v_{nj}^{e} is the relativistic correction of the electromagnetic photon emitted from the initial energy level to the final energy level and from equation (151) v_{nj}^{g} is the relativistic correction of the gravitomagnetic photon emitted from the initial energy level to the final energy level.

Multilevel operator D^{mul}

We are using Pauli matrices σ, electromagnetic four-potential A_μ and charge e with $\hbar = c = 1$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix} \quad \Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} \quad (152)$$

In four dimensions, Minkowski's metric $\eta_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$ leads to the Clifford algebra C(1,3)[9], $\{\gamma^\mu,\gamma^\nu\} = 2\eta^{\mu\nu} \times \mathbb{I}_{4\times4}$, Dirac matrices $\gamma^0 = \sigma_3 \otimes I, \gamma^j = i\sigma_2 \otimes \sigma_j, j = 1,2,3; \gamma^p = -i\gamma^{14} = -i\gamma^0\gamma^1\gamma^2\gamma^3$

$$\gamma^4 = \gamma^0\gamma^1, \gamma^5 = \gamma^0\gamma^2, \gamma^6 = \gamma^0\gamma^3, \gamma^7 = \gamma^1\gamma^2, \gamma^8 = \gamma^1\gamma^3, \gamma^9 = \gamma^2\gamma^3$$
$$\gamma^{10} = \gamma^0\gamma^1\gamma^2, \gamma^{11} = \gamma^0\gamma^1\gamma^3, \gamma^{12} = \gamma^0\gamma^2\gamma^3, \gamma^{13} = \gamma^1\gamma^2\gamma^3, \gamma^{14} = \gamma^0\gamma^1\gamma^2\gamma^3 \quad (153)$$

Multilevel operator D^n acts on level n, n is the number of γ matrices in the product of the algebra members, for example, D^3 acts on $\gamma^{10},\gamma^{11},\gamma^{12}$ and γ^{13}. Total multilevel operator $D^{\mathrm{mul}} = D^0 + D^1 + D^2 + D^3 + D^4$, the action of D^{mul} on the spinor function vanishes $D^{mul}\Psi = 0$ (154)

$$D^0 = -m \quad (155)$$

$$D^1 = \gamma^\mu p_\mu - ie\gamma^\mu A_\mu \quad (156)$$

$$D^2 = -ie\gamma^\mu\gamma^\nu F_{\mu\nu} \text{ with } F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu} \quad (157)$$

$$D^2 = -ie\alpha E + e\Sigma H \quad (158)$$

$$D^3 = -ie\gamma^\mu\gamma^\nu\gamma^\delta F_{\mu\nu\delta} \text{ with } F_{\mu\nu\delta} = A_{\mu,\nu,\delta} - A_{\mu,\delta,\nu} = 0 \quad (159)$$

$$D^4 = -ie\gamma^\mu\gamma^\nu\gamma^\delta\gamma^\lambda F_{\mu\nu\delta\lambda} \text{ with } F_{\mu\nu\delta\lambda} = A_{\mu,\nu,\delta,\lambda} - A_{\mu,\nu,\lambda,\delta} = 0 \quad (160)$$

Multilevel operator $D^{mul}(\eta_{\mu\nu}, A_\mu, e)$ can be generalized for a curved space with four-potential P, field charge q and covariant derivative [10] $(;\mu)$ instead of derivative $(,\mu)$ in the definition of p_μ

$$D^{mul}(g_{\mu\nu}, P_\mu, q)\Psi = 0 \quad (161)$$

$$\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu} \times \mathbb{I} \quad (162)$$

$$D^0 = -m \quad (163)$$

$$D^1 = \gamma^\mu p_\mu - iq\gamma^\mu P_\mu \quad (164)$$

$$D^2 = -iq\gamma^\mu\gamma^\nu G_{\mu\nu} \text{ with } G_{\mu\nu} = P_{\mu;\nu} - P_{\nu;\mu} \quad (165)$$

$$G_{\mu\nu}(P) = P_{\mu;\nu} - P_{\nu;\mu} = P_{\mu,\nu} - P_{\nu,\mu} + P_\alpha T_{\mu\nu}^\alpha \quad (166)$$

$$G_{\mu\nu}(P) = F_{\mu\nu}(P) + P_\alpha T_{\mu\nu}^\alpha \quad (167)$$

$$D^2 = -iq\alpha E(P) + q\Sigma H(P) - iq\frac{1}{2}\gamma^\mu\gamma^\nu P_\alpha T_{\mu\nu}^\alpha \quad (168)$$

For gravity $G_{\mu\nu}(P)$ is the new gravitomagnetic tensor. $T_{\mu\nu}^\alpha$ is the torsion tensor [11]

$$D^3 = -iq\gamma^\mu\gamma^\nu\gamma^\delta G_{\mu\nu\delta} \text{ with } G_{\mu\nu\delta} = P_{\mu;\nu;\delta} - P_{\mu;\delta;\nu} \quad (169)$$

$G_{\mu\nu\delta}(P) = P_\alpha R^\alpha_{\mu\nu\delta}$ with $R^\alpha_{\mu\nu\delta}$ the Riemann-Christoffel tensor[12] (170)

$$D^3 = -iq\gamma^0\gamma^1\gamma^2 P_\alpha R^\alpha_{012} - iq\gamma^0\gamma^1\gamma^3 P_\alpha R^\alpha_{013} - iq\gamma^0\gamma^2\gamma^3 P_\alpha R^\alpha_{023} - iq\gamma^1\gamma^2\gamma^3 P_\alpha R^\alpha_{123} \quad (171)$$

$D^4 = -iq\gamma^\mu\gamma^\nu\gamma^\delta\gamma^\lambda G_{\mu\nu\delta\lambda}$ with $G_{\mu\nu\delta\lambda} = P_{\mu;\nu;\delta;\lambda} - P_{\mu;\nu;\lambda;\delta}$ (172)

$$G_{\mu\nu\delta\lambda}(P) = P_{\alpha;\nu} R^\alpha_{\mu\delta\lambda} + P_{\mu;\alpha} R^\alpha_{\nu\delta\lambda} \quad (173)$$

$$D^4 = -iq\gamma^0\gamma^1\gamma^2\gamma^3 P_{\alpha;1} R^\alpha_{023} - iq\gamma^0\gamma^1\gamma^2\gamma^3 P_{0;\alpha} R^\alpha_{123} \quad (174)$$

Gravitomagnetic tensor defined in Schwarzschild's metric

We are using $x^0 = t, x^1 = r, x^2 = \theta, x^3 = \phi$ with $G = c = 1$, this metric is defined by [13]

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \quad (175)$$

$$g_{00} = \left(1 - \frac{2M}{r}\right), g_{11} = -\left(1 - \frac{2M}{r}\right)^{-1}, g_{22} = -r^2, g_{33} = -r^2 \sin^2\theta \quad (176)$$

$$g^{00} = \left(1 - \frac{2M}{r}\right)^{-1}, g^{11} = -\left(1 - \frac{2M}{r}\right), g^{22} = -r^{-2}, g^{33} = -r^{-2}\sin^{-2}\theta \quad (177)$$

$$\Gamma^1_{00} = g_{00} M r^{-2} \quad (178)$$

$$\Gamma^0_{01} = g_{00}^{-1} M r^{-2} \quad (179)$$

$$\Gamma^1_{11} = -g_{00}^{-1} M r^{-2} \quad (180)$$

$$\Gamma^2_{12} = \Gamma^3_{13} = r^{-1} \quad (181)$$

$$\Gamma^1_{22} = -g_{00} r \quad (182)$$

$$\Gamma^3_{23} = \cot\theta \quad (183)$$

$$\Gamma^1_{33} = -g_{00} r \sin^2\theta \quad (184)$$

$$\Gamma^2_{33} = -\sin\theta\cos\theta \quad (185)$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \times \mathbb{I} \quad (186)$$

$$\gamma^0 = g_{00}^{-1/2} \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (187)$$

$$\gamma^1 = -g_{00}^{1/2} \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix} \quad (188)$$

$$\gamma^2 = -r^{-1} \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} \quad (190)$$

$$\gamma^3 = -r^{-1}\sin^{-1}\theta \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix} \quad (191)$$

$D^2 = im\alpha E(P) - m\Sigma H(P) + im\frac{1}{2}\gamma^\mu\gamma^\nu P_\alpha T^\alpha_{\mu\nu}$, from equations (167) and (168)

$$G_{12} = -H_3(P) = \left(-g_{00}^{1/2}\right)(-r^{-1})(P_{1,2} - P_{2,1}) \quad (192)$$

$$G_{13} = H_2(P) = \left(-g_{00}^{1/2}\right)(-r^{-1}\sin^{-1}\theta)(P_{1,3} - P_{3,1}) \quad (193)$$

$$G_{23} = -H_1(P) = (-r^{-1})(-r^{-1}\sin^{-1}\theta)(P_{2,3} - P_{3,2}) \quad (194)$$

$$G_{03} = -E_3(P) = \left(g_{00}^{-1/2}\right)(-r^{-1}\sin^{-1}\theta)(P_{0,3} - P_{3,0}) \quad (195)$$

$$G_{02} = -E_2(P) = \left(g_{00}^{-1/2}\right)(-r^{-1})(P_{0,2} - P_{2,0}) \quad (196)$$

$$G_{01} = -E_1(P) = \left(g_{00}^{-1/2}\right)\left(-g_{00}^{1/2}\right)\left(P_{0,1} - P_{1,0}\right) \quad (197)$$

$T^{\alpha}_{\mu\nu} = 0$ and $R^{\alpha}_{012} = R^{\alpha}_{013} = R^{\alpha}_{023} = R^{\alpha}_{123} = 0$ (198)

Energy-momentum form is a 1 -form [14]

$$\mathbf{p} = E\mathbf{d}t - p_x\mathbf{d}x - p_y\mathbf{d}y - p_z\mathbf{d}z \quad (199)$$

$\mathbf{dp}$ is a 2 -form

$$\mathbf{G} = \mathbf{dp} = E_x\mathbf{d}t \wedge \mathbf{d}x + E_y\mathbf{d}t \wedge \mathbf{d}y + E_z\mathbf{d}t \wedge \mathbf{d}z - B_x\mathbf{d}y \wedge \mathbf{d}z - B_y\mathbf{d}z \wedge \mathbf{d}x - B_z\mathbf{d}x \wedge \mathbf{d}y \quad (200)$$

$$G_{32} = p_{z,y} - p_{y,z} \quad (201)$$

$$G_{13} = p_{x,z} - p_{z,x} \quad (202)$$

$$G_{21} = p_{y,x} - p_{x,y} \quad (203)$$

Comparing equations (192-194) and (201-203) we can infer $P_{\alpha} = p_{\alpha}$ (204)

D^0 is related to the scalar 0-form m, D^1 is related to the Energy-momentum 1 -form, D^2 is related to the Electromagnetic 2 -form, D^3 is related to $*\,\mathbf{J}$3-form [15]

$$\begin{pmatrix} *J_{123} \\ *J_{023} \\ *J_{013} \\ *J_{012} \end{pmatrix} = \begin{pmatrix} -\rho \\ j_1 \\ -j_2 \\ j_3 \end{pmatrix} \quad (205)$$

D^4 is related to $\mathbf{L}$4-form [16]

$$\mathbf{L} = L_{0123}\mathbf{d}x^0 \wedge \mathbf{d}x^1 \wedge \mathbf{d}x^2 \wedge \mathbf{d}x^3 \quad (206)$$

$\gamma^p = -i\gamma^{14}$ is the proyector matrix, historically γ^5, but $\gamma^5 = \gamma^0\gamma^2$

$$\gamma^p = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \left(g_{00}^{-1/2}\right)\left(-g_{00}^{1/2}\right)(-r^{-1})(-r^{-1}\sin^{-1})\begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad (207)$$

Torsion tensor in a rearranged Kerr's metric

We are using $x^0 = t, x^1 = r, x^2 = \theta, x^3 = \phi$, M is the black hole's mass and a is the angular momentum per unit mass with $G = c = 1$. The invariance of the length of vectors under parallel transport means that the connection is compatible with the metric, it is a metric connection, the requirement of the preservation of the length by parallel transport may be stated as [17]

$$g_{\mu\nu;\sigma} = 0 \quad (208)$$

$$g_{\mu\nu;\sigma} = g_{\mu\nu,\sigma} - g_{\alpha\nu}\Gamma^{\alpha}_{\mu\sigma} - g_{\mu\alpha}\Gamma^{\alpha}_{\nu\sigma} \quad (209)$$

$0 = g_{\mu\nu,\sigma} - g_{\nu\alpha}\Gamma^{\alpha}_{\mu\sigma} - t_{\alpha\nu}\Gamma^{\alpha}_{\mu\sigma} - g_{\mu\alpha}\Gamma^{\alpha}_{\nu\sigma}$, with $t_{\mu\nu} = g_{\mu\nu} - g_{\nu\mu}$ (210)

$$0 = g_{\mu\nu,\sigma} - g_{\nu\alpha}\Gamma^{\alpha}_{\mu\sigma} - t_{\alpha\nu}\Gamma^{\alpha}_{\mu\sigma} - g_{\mu\alpha}\Gamma^{\alpha}_{\nu\sigma} \quad (211)$$

$$g_{\mu\alpha}\Gamma^{\alpha}_{\nu\sigma} + g_{\nu\alpha}\Gamma^{\alpha}_{\mu\sigma} + t_{\alpha\nu}\Gamma^{\alpha}_{\mu\sigma} = g_{\mu\nu,\sigma} \quad (212)$$

$$\Gamma_{\mu\nu\sigma} + \Gamma_{\nu\mu\sigma} + t_{\alpha\nu}g^{\alpha\lambda}\Gamma_{\lambda\mu\sigma} = g_{\mu\nu,\sigma} \quad (213)$$

Solving these equations we get the torsion applying its definition [18]

$$\Gamma_{\mu\nu\sigma} - \Gamma_{\mu\sigma\nu} = -T_{\mu\nu\sigma} \quad (214)$$

Expanding the line element in powers of r^{-1} and examining the leading terms [19]

$$ds^2 = \left[1 - \frac{2M}{r} + O(r^{-2})\right]dt^2 + \left[\frac{4aM}{r}\sin^2\theta + O(r^{-2})\right]dtd\phi - [1 + O(r^{-1})][dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2] \quad (215)$$

Rearranging the line elements

$$ds^2 = \left[1 - \frac{2M}{r} + O(r^{-2})\right]dt^2 + [O(r^{-2})]dtd\phi + \left[\frac{4aM}{r}\sin^2\theta\right]d\phi dt - [1 + O(r^{-1})][dr^2 +$$

$$r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad (216)$$

$$g_{00} = \left[1 - \frac{2M}{r} + O(r^{-2})\right] \quad (217)$$

$$g_{03} = [O(r^{-2})] \quad (218)$$

$$g_{11} = -[1 + O(r^{-1})] \quad (219)$$

$$g_{22} = -[1 + O(r^{-1})]r^2 \quad (220)$$

$$g_{33} = -[1 + O(r^{-1})]r^2 \sin^2\theta \quad (221)$$

$$g_{30} = \left[\frac{4aM}{r}\sin^2\theta\right] \quad (222)$$

$$t_{03} = [O(r^{-2})] - \left[\frac{4aM}{r}\sin^2\theta\right] \quad (223)$$

$$g^{00} = g_{33}(g_{00}g_{33} - g_{03}g_{30})^{-1} \quad (224)$$

$$g^{03} = -g_{03}(g_{00}g_{33} - g_{03}g_{30})^{-1} \quad (225)$$

$$g^{11} = g_{11}^{-1} \quad (226)$$

$$g^{22} = g_{22}^{-1} \quad (227)$$

$$g^{30} = -g_{30}(g_{00}g_{33} - g_{03}g_{30})^{-1} \quad (228)$$

$$g^{33} = g_{00}(g_{00}g_{33} - g_{03}g_{30})^{-1} \quad (229)$$

Generalizing Clifford algebra with $\delta_{\mu\nu}^{clif}$,if $\mu \neq \nu$ and $g_{\mu\nu} \neq 0$ then 1 ,else 0

$$\{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu} + g^{\nu\mu} - g^{\mu\nu}\delta_{\mu\nu}^{clif} - g^{\nu\mu}\delta_{\nu\mu}^{clif} \quad (230)$$

$$\gamma^0 = [1 + O(r^{-1})]^{1/2} r\sin\theta (g_{00}g_{33} - g_{03}g_{30})^{-1/2}\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (231)$$

$$\gamma^1 = [1 + O(r^{-1})]^{-1/2}\begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix} \quad (232)$$

$$\gamma^2 = [1 + O(r^{-1})]^{-1/2} r^{-1}\begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} \quad (233)$$

$$\gamma^3 = \left[1 - \frac{2M}{r} + O(r^{-2})\right]^{1/2}(g_{00}g_{33} - g_{03}g_{30})^{-1/2}\begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix} \quad (234)$$

$D^2 = im\alpha E(P) - m\Sigma H(P) + im\frac{1}{2}\gamma^\mu\gamma^\nu P_\alpha T_{\mu\nu}^\alpha$, from equations (167) and (168) (235)

$$F_{12} = -H_3(P) = \left([1 + O(r^{-1})]^{-1/2}\right)\left([1 + O(r^{-1})]^{-1/2} r^{-1}\right)(P_{1,2} - P_{2,1}) \quad (236)$$

$$F_{13} = H_2(P) = \left([1 + O(r^{-1})]^{-1/2}\right)\left(\left[1 - \frac{2M}{r} + O(r^{-2})\right]^{1/2}(g_{00}g_{33} - g_{03}g_{30})^{-1/2}\right)(P_{1,3} - P_{3,1}) \quad (237)$$

$$F_{23} = -H_1(P) = \left([1 + O(r^{-1})]^{-1/2} r^{-1}\right)\left(\left[1 - \frac{2M}{r} + O(r^{-2})\right]^{1/2}(g_{00}g_{33} - g_{03}g_{30})^{-1/2}\right)(P_{2,3} - P_{3,2}) \quad (238)$$

$$F_{03} = -E_3(P) = \left([1 + O(r^{-1})]^{1/2} r\sin\theta (g_{00}g_{33} - g_{03}g_{30})^{-1/2}\right)\left(\left[1 - \frac{2M}{r} + O(r^{-2})\right]^{1/2}(g_{00}g_{33} - g_{03}g_{30})^{-1/2}\right)(P_{0,3} - P_{3,0}) \quad (239)$$

$$F_{02} = -E_2(P) = \left([1 + O(r^{-1})]^{1/2} r\sin\theta (g_{00}g_{33} - g_{03}g_{30})^{-1/2}\right)\left([1 + O(r^{-1})]^{-1/2} r^{-1}\right)(P_{0,2} - P_{2,0}) \quad (240)$$

$$F_{01} = -E_1(P) = \left([1 + O(r^{-1})]^{1/2} r\sin\theta (g_{00}g_{33} - g_{03}g_{30})^{-1/2}\right)\left([1 + O(r^{-1})]^{-1/2}\right)(P_{0,1} - P_{1,0}) \quad (241)$$

$\gamma^p = -i\gamma^{14}$ is the proyector matrix, historically γ^5, but $\gamma^5 = \gamma^0\gamma^2$

$$\gamma^p = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \left([1+O(r^{-1})]^{1/2} r\sin\theta (g_{00}g_{33} - g_{03}g_{30})^{-1/2}\right)\left([1+O(r^{-1})]^{-1/2}\right)\left([1+ O(r^{-1})]^{-1/2} r^{-1}\right)\left(\left[1-\frac{2M}{r}+O(r^{-2})\right]^{1/2} (g_{00}g_{33}-g_{03}g_{30})^{-1/2}\right)\begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad (242)$$

Geodesic equation and torsion tensor

A geodesic that is not a null geodesic has the property that $\int ds$, taken along a section of the track with the end points P and Q, is stationary if one makes a small variation of the track keeping the end points fixed. If dx^μ denotes an element along the track [20]

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \quad (243)$$

$$2ds\delta(ds) = dx^\mu dx^\nu \delta(g_{\mu\nu}) + g_{\mu\nu}\delta(dx^\mu)dx^\nu + g_{\mu\nu}dx^\mu\delta(dx^\nu) \quad (244)$$

$$2ds\delta(ds) = dx^\mu dx^\nu g_{\mu\nu,\lambda}\delta(x^\lambda) + 2g_{\mu\lambda}dx^\mu\delta(dx^\lambda) \quad (245)$$

$$\delta(dx^\lambda) = d\delta(x^\lambda) \text{ and } dx^\mu = v^\mu ds \quad (246)$$

$$\int \delta(ds) = \int \left[\frac{1}{2}g_{\mu\nu,\lambda}v^\mu v^\nu \delta x^\lambda + g_{\mu\lambda}v^\mu \frac{d\delta x^\lambda}{ds}\right] ds \quad (247)$$

By partial integration with $\delta x^\lambda = 0$ at end points P and Q, we get

$$\delta\int ds = \int \left[\frac{1}{2}g_{\mu\nu,\lambda}v^\mu v^\nu - \frac{d}{ds}(g_{\mu\lambda}v^\mu)\right]\delta x^\lambda ds \quad (248)$$

The condition for this to vanish with arbitrary δx^λ is

$$\frac{d}{ds}(g_{\mu\lambda}v^\mu) - \frac{1}{2}g_{\mu\nu,\lambda}v^\mu v^\nu = 0 \quad (249)$$

$$\frac{d}{ds}(g_{\mu\lambda}v^\mu) = g_{\mu\lambda}\frac{dv^\mu}{ds} + g_{\mu\lambda,\nu}v^\mu v^\nu \quad (250)$$

$$\frac{d}{ds}(g_{\mu\lambda}v^\mu) = g_{\lambda\mu}\frac{dv^\mu}{ds} - t_{\lambda\mu}\frac{dv^\mu}{ds} + \frac{1}{2}(g_{\mu\lambda,\nu} + g_{\nu\lambda,\mu})v^\mu v^\nu \text{, and with } t_{\mu\lambda} = g_{\mu\lambda} - g_{\lambda\mu} \quad (251)$$

$$\frac{d}{ds}(g_{\mu\lambda}v^\mu) = g_{\lambda\mu}\frac{dv^\mu}{ds} - t_{\lambda\mu}\frac{dv^\mu}{ds} + \frac{1}{2}(g_{\lambda\mu},\nu + g_{\lambda\nu},\mu)v^\mu v^\nu + \frac{1}{2}(t_{\mu\lambda,\nu} + t_{\nu\lambda,\mu})v^\mu v^\nu \quad (252)$$

From equation (249) with $t_{\mu\nu,\lambda} = g_{\mu\nu,\lambda} - g_{\nu\mu,\lambda}$

$$\frac{d}{ds}(g_{\mu\lambda}v^\mu) - \frac{1}{2}g_{\nu\mu,\lambda}v^\mu v^\nu - \frac{1}{2}t_{\mu\nu},\lambda v^\mu v^\nu = 0 \quad (253)$$

Thus the condition (253) becomes

$$g_{\lambda\mu}\frac{dv^\mu}{ds} - t_{\lambda\mu}\frac{dv^\mu}{ds} + \bar{\Gamma}_{\lambda\mu\nu}v^\mu v^\nu - \dot{\Gamma}_{\lambda\mu\nu}v^\mu v^\nu = 0 \quad (254)$$

$$\bar{\Gamma}_{\lambda\mu\nu} = \frac{1}{2}(g_{\lambda\mu,\nu} + g_{\lambda\nu},\mu - g_{\nu\mu,\lambda}) \quad (255)$$

$$\dot{\Gamma}_{\lambda\mu\nu} = \frac{1}{2}(t_{\lambda\mu,\nu} + t_{\lambda\nu,\mu} - t_{\nu\mu,\lambda}) \quad (256)$$

Multiplying equation (254) by $g^{\sigma\lambda}$, we obtain the geodesic equation

$$\frac{dv^\sigma}{ds} - g^{\sigma\lambda}t_{\lambda\mu}\frac{dv^\mu}{ds} + \bar{\Gamma}^\sigma_{\mu\nu}v^\mu v^\nu - \dot{\Gamma}^\sigma_{\mu\nu}v^\mu v^\nu = 0 \quad (257)$$

$$\frac{dv^\sigma}{ds} - g^{\sigma\lambda}t_{\lambda\mu}\frac{dv^\mu}{ds} + \Gamma^\sigma_{\mu\nu}v^\mu v^\nu = 0 \quad (258)$$

$$\Gamma^\sigma_{\mu\nu} = \bar{\Gamma}^\sigma_{\mu\nu} - \dot{\Gamma}^\sigma_{\mu\nu} \quad (259)$$

$\bar{\Gamma}^\sigma_{\mu\nu}$ are the Christoffel symbols of the symmetric part, so

$$-\bar{T}^\sigma_{\mu\nu} = \bar{\Gamma}^\sigma_{\mu\nu} - \bar{\Gamma}^\sigma_{\nu\mu} = \Gamma^\sigma_{\mu\nu} - \Gamma^\sigma_{\nu\mu} + \dot{\Gamma}^\sigma_{\mu\nu} - \dot{\Gamma}^\sigma_{\nu\mu} \quad (260)$$

$$0 = \Gamma^\sigma_{\mu\nu} - \Gamma^\sigma_{\nu\mu} + \dot{\Gamma}^\sigma_{\mu\nu} - \dot{\Gamma}^\sigma_{\nu\mu} \quad (261)$$

We directly obtain the torsion tensor without solving equations (213) and (214)

$$T_{\mu\nu}^{\sigma} = \dot{\Gamma}_{\mu\nu}^{\sigma} - \dot{\Gamma}_{\nu\mu}^{\sigma} \quad (262)$$

Einstein field equation and conservation laws

From equation (259) where $\bar{\Gamma}_{\mu\nu}^{\sigma}$ are the symbols of the symmetric part

$$\Gamma_{\nu\mu}^{\mu} = \bar{\Gamma}_{\nu\mu}^{\mu} - \dot{\Gamma}_{\nu\mu}^{\mu} \quad (263)$$

$$\bar{\Gamma}_{\nu\mu}^{\mu} = \bar{\Gamma}_{\mu\nu}^{\mu} = g^{\mu\lambda}\bar{\Gamma}_{\lambda\mu\nu} = g^{\mu\lambda}\frac{1}{2}\left(g_{\lambda\mu,\nu} + g_{\lambda\nu,\mu} - g_{\nu\mu,\lambda}\right) \quad (264)$$

$$\bar{\Gamma}_{\nu\mu}^{\mu} = \left(\sqrt{-g}\right)^{-1}\sqrt{-g}_{,\nu} + g^{\mu\lambda}\frac{1}{2}\left(g_{\lambda\nu,\mu} - g_{\nu\mu,\lambda}\right) \quad (265)$$

Equation (263) becomes

$$\Gamma_{\nu\mu}^{\mu} = \left(\sqrt{-g}\right)^{-1}\sqrt{-g}_{,\nu} + g^{\mu\lambda}\frac{1}{2}\left(g_{\lambda\nu,\mu} - g_{\nu\mu,\lambda}\right) - \dot{\Gamma}_{\nu\mu}^{\mu} \quad (266)$$

$$\Gamma_{\nu\mu}^{\mu} = \left(\sqrt{-g}\right)^{-1}\sqrt{-g}_{,\nu} - t_{\nu} \quad (267)$$

$$t_{\nu} = -g^{\mu\lambda}\frac{1}{2}\left(g_{\lambda\nu},\mu - g_{\nu\mu},\lambda\right) + \dot{\Gamma}_{\nu\mu}^{\mu} \quad (268)$$

The vector A^{μ} has the covariant divergence

$$A_{;\mu}^{\mu} = A_{,\mu}^{\mu} + \Gamma_{\nu\mu}^{\mu}A^{\nu} \quad (269)$$

$$\left(A_{;\mu}^{\mu} + t_{\nu}A^{\nu}\right)\sqrt{-g} = \left(A^{\mu}\sqrt{-g}\right)_{,\mu} \quad (270)$$

If the left-hand side of equation (270) equals zero then the right-hand side gives us the first conservation law.

For the antisymmetric tensor $F^{\mu\nu} = -F^{\nu\mu}$

$$F_{;\nu}^{\mu\nu} = F^{\mu\nu}{}_{,\nu} + \Gamma_{\rho\nu}^{\mu}F^{\rho\nu} + \Gamma_{\rho\nu}^{\nu}F^{\mu\rho} \quad (271)$$

$$F_{;\nu}^{\mu\nu} = F_{,\nu}^{\mu\nu} - T_{\rho\nu}^{\mu}F^{\rho\nu} + \left(\left(\sqrt{-g}\right)^{-1}\sqrt{-g}_{,\rho} - t_{\rho}\right)F^{\mu\rho} \quad (272)$$

$$\left(F_{;\nu}^{\mu\nu} + T_{\rho\nu}^{\mu}F^{\rho\nu} + t_{\rho}F^{\mu\rho}\right)\sqrt{-g} = \left(F^{\mu\nu}\sqrt{-g}\right)_{,\nu} \quad (273)$$

If the left-hand side of equation (273) equals zero then the right-hand side gives us the second conservation law.

For the antisymmetric tensor $F_{\mu\nu} = A_{\mu;\nu} - A_{\nu;\mu}$

$$F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu} + T_{\mu\nu}^{\alpha}A_{\alpha} \quad (274)$$

$$F_{\mu\nu;\sigma} = F_{\mu\nu,\sigma} - \Gamma_{\mu\sigma}^{\alpha}F_{\alpha\nu} - \Gamma_{\nu\sigma}^{\alpha}F_{\mu\alpha} \quad (275)$$

$$F_{\nu\sigma;\mu} = F_{\nu\sigma,\mu} - \Gamma_{\nu\mu}^{\alpha}F_{\alpha\sigma} - \Gamma_{\sigma\mu}^{\alpha}F_{\nu\alpha} \quad (276)$$

$$F_{\sigma\mu;\nu} = F_{\sigma\mu,\nu} - \Gamma_{\sigma\nu}^{\alpha}F_{\alpha\mu} - \Gamma_{\mu\nu}^{\alpha}F_{\sigma\alpha} \quad (277)$$

Adding equations (275), (276) and (277)

$$F_{\mu\nu;\sigma} + F_{\nu\sigma;\mu} + F_{\sigma\mu;\nu} = T_{\mu\nu}^{\alpha}F_{\sigma\alpha} + \left(T_{\mu\nu}^{\alpha}A_{\alpha}\right)_{,\sigma} + T_{\nu\sigma}^{\alpha}F_{\mu\alpha} + \left(T_{\nu\sigma}^{\alpha}A_{\alpha}\right)_{,\mu} + T_{\sigma\mu}^{\alpha}F_{\nu\alpha} + \left(T_{\sigma\mu}^{\alpha}A_{\alpha}\right)_{,\nu} \quad (278)$$

From the definition of the curvature tensor $R_{\nu\rho\sigma}^{\beta}$

$$R_{\nu\rho\sigma}^{\beta} = \Gamma_{\nu\sigma,\rho}^{\beta} - \Gamma_{\nu\rho,\sigma}^{\beta} + \Gamma_{\nu\sigma}^{\alpha}\Gamma_{\alpha\rho}^{\beta} - \Gamma_{\nu\rho}^{\alpha}\Gamma_{\alpha\sigma}^{\beta} \quad (279)$$

$R_{\nu\mu\rho}^{\mu}$ is called the Ricci tensor

$$R_{\mu\nu} = -\Gamma_{\mu\alpha,\nu}^{\alpha} + \Gamma_{\mu\nu,\alpha}^{\alpha} - \Gamma_{\mu\beta}^{\alpha}\Gamma_{\alpha\nu}^{\beta} + \Gamma_{\mu\nu}^{\alpha}\Gamma_{\alpha\beta}^{\beta} \quad (280)$$

Now $R_{\mu\nu}$ is not symmetric, $2A_{\mu\nu} = R_{\mu\nu} - R_{\nu\mu}$ is the antisymmetric part and $R_{\mu\nu} = A_{\mu\nu} + S_{\mu\nu}$ where $S_{\mu\nu}$ is the symmetric part in the Einstein's equation [21].

$$S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S = \kappa T_{\mu\nu} \tag{281}$$

$$2A_{\mu\nu} = -((\sqrt{-g})^{-1}\sqrt{-g}_{,\mu} - t_\mu)_{,\nu} + ((\sqrt{-g})^{-1}\sqrt{-g}_{,\nu} - t_\nu)_{,\mu} - T^{\alpha}_{\mu\nu,\alpha} - T^{\alpha}_{\mu\nu}((\sqrt{-g})^{-1}\sqrt{-g}_{,\alpha} - t_\alpha) \tag{282}$$

$$2A_{\mu\nu} = t_{\mu,\nu} - t_{\nu,\mu} - T^{\alpha}_{\mu\nu,\alpha} - T^{\alpha}_{\mu\nu}\left((\sqrt{-g})^{-1}\sqrt{-g}_{,\alpha} - t_\alpha\right) \tag{283}$$

$$R_{\mu\nu} - A_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(R - A) = \kappa T_{\mu\nu} \tag{284}$$

$$R_{\mu\nu} - A_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} \tag{285}$$

Conclusions

Multilevel operator $D^{mul}\left(g_{\mu\nu}, P_\mu, q\right)$ has been generalized for a curved space with a general four-potential P. For gravity $G_{\mu\nu}(P)$ is the new gravitomagnetic tensor and torsion tensor $T^{\alpha}_{\mu\nu}$ appears in its definition

In a flat space $G_{\mu\nu}(A) = F_{\mu\nu}(A)$, D^3 and D^4 operators vanish. In a curved space the curvature tensor $R^{\alpha}_{\mu\nu\delta}$ appears in levels 3 and 4

The appearance of torsion tensor $T^{\alpha}_{\mu\nu}$ and curvature tensor $R^{\alpha}_{\mu\nu\delta}$ in multilevel operator $D^{mul}\left(g_{\mu\nu}, P_\mu, q\right)$ means that this operator is a fundamental operator in Quantum Field Theory

$\gamma^0, \gamma^1, \gamma^2, \gamma^3$, have been calculated for Schwarzschild's metric, then $G_{\mu\nu}(P)$, the gravitomagnetic tensor has been obtained

Each D^n, where n is the number of γ matrices in the product of the algebra members, is related to an n-form

The invariance of the length of vectors under parallel transport requires the vanishing of the metric tensor covariant derivative, a new term appears $t_{\mu\nu} = g_{\mu\nu} - g_{\nu\mu}$ measuring the non symmetric part of the metric tensor, solving these equations we get the torsion tensor

Rearranging Kerr's metric we obtained $t_{03} = g_{03} - g_{30}$, the non symmetric part of the metric tensor, gravitomagnetic tensor has also been calculated generalizing the Clifford algebra

Taking into account the $t_{\mu\nu} = g_{\mu\nu} - g_{\nu\mu}$ in the geodesic equation we have obtained the torsion tensor, conservation laws and Einstein field equation in a non-symmetric geometry

The solution of Schrödinger's equation leads us to the emission of gravitomagnetic photons when the gravitational potential is also taken into account. The correction of the second term in electromagnetic photon frequency is an indirect detection of the gravitomagnetic photon emission.

The solution of Schrödinger's equation for a gravitational system leads us to Planck's gravitational constant value and Kepler's third law.

New broken symmetries lead us to gravity and nuclear force unification and new short distance forces, new Higgs bosons, intermediate vector bosons and photons appear. Planck's constant is a parameter that must be calculated for different systems.

In rainbow unification we have electromagnetic, gravitomagnetic and colormagnetic forces with infinite and short distance interactions. The gravitomagnetic Higgs boson detection will confirm the existence of the gravitomagnetic photon and the final version of the Standard Model of particle physics.

References

1. Marian Fecko (2006) Differential Geometry and Lie Groups for Physicists. *Cambridge University Press* 637.
2. D.H. Sattinger., O.L.Weaver (1986) Lie Groups and Algebras with Applications to Physics, Geometry and Mechanics. *Springer-Verlag New York Inc, Ann Arbor* 169
3. Kurt Gottfried., Victor F. Weisskopf (1984) Concepts of Particle Physics, Volume I. *Oxford University Press* 81.
4. Robert Eisberg., Robert Resnick (1979) Fisica Cuantica. Atomos, moleculas, solidos, nucleos y particulas. *Editorial Limusa* 286.
5. Robert Eisberg., Robert Resnick (1979) Fisica Cuantica. Atomos, moleculas, solidos, nucleos y particulas. *Editorial Limusa* 281.

6. Robert Eisberg., Robert Resnick (1979) Fisica Cuantica. Atomos, moleculas, solidos, nucleos y particulas. *Editorial Limusa* 337.
7. Delso J (2022) On Allais Effect Explained by The Gravitomagnetic Tensor. *OSP Journal of Physics and Astronomy* 3.
8. Delso J (2021) On Gravitational Magnetic Field Derived from Special Relativity Leading to Dark Matter Force. *OSP J Phy Astronomy* 2.
9. Marian Fecko (2006) Differential Geometry and Lie Groups for Physicists. *Cambridge University Press* 650-652.
10. Marian Fecko (2006) Differential Geometry and Lie Groups for Physicists. *Cambridge University Press* 380.
11. Marian Fecko (2006) Differential Geometry and Lie Groups for Physicists. *Cambridge University Press* 384
12. Marian Fecko (2006) Differential Geometry and Lie Groups for Physicists. *Cambridge University Press* 404.
13. P.A.M. Dirac (1996) General Theory of Relativity. *Princeton University Press* 30-32.
14. J.A. Wheeler., C. Misner., K.S. Thorne (2017) Gravitation. *Princeton University Press* 91.
15. Roger Penrose (2006) El camino a la realidad Random House Mondadori. *Barcelona* 603.
16. J.A. Wheeler., C. Misner., K.S. Thorne (2017) Gravitation. *Princeton University Press* 119.
17. Marian Fecko (2006) Differential Geometry and Lie Groups for Physicists. *Cambridge University Press* 383.
18. Marian Fecko (2006) Differential Geometry and Lie Groups for Physicists. *Cambridge University Press* 389.
19. J.A. Wheeler., C. Misner., K.S. Thorne (2017) Gravitation. *Princeton University Press* 891.
20. P.A.M. Dirac (1996) General Theory of Relativity. *Princeton University Press* 16-17.
21. J.A. Wheeler., C. Misner., K.S. Thorne (2017) Gravitation. Princeton University Press 406.