

On Torsion Tensor in a Rearranged Kerr's Metric with a Generalized Clifford Algebra

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Abstract

In four dimensions, the Minkowski metric $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ leads to the 16-dimensional Clifford algebra $C(1,3)$, Dirac's equation [1] is using four of these 16 matrices that form a basis of this algebra, a new operator is defined using all of these matrices and also generalized for a curved space. This new multilevel operator generalizes the Dirac's equation, the value of the generalized Dirac's operator is calculated in the Schwarzschild's metric. The torsion tensor is calculated taking into account the non-symmetric part of the metric tensor in the vanishing of its covariant derivative and applied to Kerr's metric generalizing the Clifford algebra.

Keywords: *Gravitomagnetic Tensor; Gravitational Magnetic Field; Energy-momentum-Form; Clifford Algebra; Dirac Equation; Dirac Operator; Gravity and Quantum Mechanics Unification; Multilevel Operator; Schwarzschild's Metric; Torsion Tensor; Rearranged Kerr's Metric; Generalized Clifford Algebra*

Introduction

Dirac's equation is the relativistic wave equation derived by physicist Paul Dirac in 1928. The wave functions in the Dirac theory are vectors of four complex components (known as bispinors), two of which resemble the Pauli wavefunction in the non-relativistic limit, in contrast to the Schrodinger equation which described wave functions of only one complex component [1, 2].

Dirac's operator is just the tip of the iceberg, the tip of a generalized operator that is obtained by operating on all members of the Clifford algebra basis and not just on four of them.

The Schwarzschild's metric is named in honour of Karl Schwarzschild, who found the exact solution in 1915 and published it in January 1916, a little more than a month after the publication of Einstein's theory of general relativity. It was the first exact solution of the Einstein field equations other than the trivial at space solution. Schwarzschild died shortly after his paper was published, as a result of a disease he developed while serving in the German army during World War I. Johannes Droste in 1916 independently produced the same solution as Schwarzschild.

Schwarzschild's metric is an exact solution to the Einstein's field equations that describes the gravitational field outside a spherical mass, on the assumption that the electric charge of the mass, angular momentum of the mass, and universal cosmological constant is all zero.

The new generalized Dirac's operator, the multilevel operator, is calculated in the Schwarzschild's metric, torsion

tensor and new gravitomagnetic tensor appear in level 2, curvature tensor appears in levels 3 and 4.

The Kerr's metric is a generalization to a rotating body of the Schwarzschild's metric.

Multilevel operator D^{mul}

We are using Pauli matrices σ , electromagnetic four-potential A_μ and charge e with $\hbar=c=1$

$$\begin{aligned} I &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \sigma_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \sigma_2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & \sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ \alpha &= \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix} & \Sigma &= \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} & & & & 1 \end{aligned}$$

In four dimensions, Minkowski's metric $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ leads to the Clifford algebra $C(1,3)$ [2], $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \times I_{4 \times 4}$, Dirac matrices $\gamma^0 = \sigma_3 \otimes I$, $\gamma^j = i\sigma_2 \otimes \sigma_j$, $j=1,2,3$; $\gamma^\mu = -i\gamma^{14} = -i\gamma^0 \gamma^1 \gamma^2 \gamma^3$

$$\begin{aligned} \gamma^4 &= \gamma^0 \gamma^1, \gamma^5 = \gamma^0 \gamma^2, \gamma^6 = \gamma^0 \gamma^3, \gamma^7 = \gamma^1 \gamma^2, \gamma^8 = \gamma^1 \gamma^3, \gamma^9 = \gamma^2 \gamma^3 \\ \gamma^{10} &= \gamma^0 \gamma^1 \gamma^2, \gamma^{11} = \gamma^0 \gamma^1 \gamma^3, \gamma^{12} = \gamma^0 \gamma^2 \gamma^3, \gamma^{13} = \gamma^1 \gamma^2 \gamma^3, \gamma^{14} = \gamma^0 \gamma^1 \gamma^2 \gamma^3 \end{aligned} \quad 2$$

Multilevel operator D^n acts on level n , n is the number of matrices in the product of the algebra members, for example, D^3 acts on $\gamma^{10}, \gamma^{11}, \gamma^{12}$ and γ^{13} . Total multilevel operator $D^{mul} = D^0 + D^1 + D^2 + D^3 + D^4$, the action of D^{mul} on the spinor function vanishes $D^{mul} \Psi = 0$

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$$D^0 = -m \quad 4 \quad G_{\mu\nu\delta\lambda}(P) = P_{\alpha;\nu}R_{\mu\delta\lambda}^\alpha + P_{\mu;\alpha}R_{\nu\delta\lambda}^\alpha \quad 21$$

$$D^1 = \gamma^\mu p_\mu - ie\gamma^\mu A_\mu \quad 5 \quad D^4 = -iq\gamma^0\gamma^1\gamma^2\gamma^3 P_{\alpha;1}R_{023}^\alpha - iq\gamma^0\gamma^1\gamma^2\gamma^3 P_{0;\alpha}R_{123}^\alpha \quad 22$$

$$D^2 = -ie\gamma^\mu\gamma^\nu F_{\mu\nu} \text{ with } F_{\mu\nu} = A_{\mu;\nu} - A_{\nu;\mu} \quad 6$$

$$D^2 = -ie\alpha E + e\Sigma H \quad 7$$

$$D^3 = -ie\gamma^\mu\gamma^\nu\gamma^\delta F_{\mu\nu\delta} \text{ with } F_{\mu\nu\delta} = A_{\mu;\nu;\delta} - A_{\mu;\delta;\nu} = 0 \quad 8$$

$$D^4 = -ie\gamma^\mu\gamma^\nu\gamma^\delta\gamma^\lambda F_{\mu\nu\delta\lambda} \text{ with } F_{\mu\nu\delta\lambda} = A_{\mu;\nu;\delta;\lambda} - A_{\mu;\nu;\lambda;\delta} = 0 \quad 9$$

Multilevel operator $D^{mul}(\eta_{\mu\nu}, A_\mu, e)$ can be generalized for a curved space with four-potential P, field charge q and covariant derivative [3] $(\cdot_{;\mu})$ instead of derivative $(\cdot_{,\mu})$ in the definition of p_μ

$$D^{mul}(g_{\mu\nu}, P_\mu, q)\Psi = 0 \quad 10$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \times \mathbb{I} \quad 11$$

$$D^0 = -m \quad 12$$

$$D^1 = \gamma^\mu p_\mu - iq\gamma^\mu P_\mu \quad 13$$

$$D^2 = -iq\gamma^\mu\gamma^\nu G_{\mu\nu} \text{ with } G_{\mu\nu} = P_{\mu;\nu} - P_{\nu;\mu} \quad 14$$

$$G_{\mu\nu}(P) = P_{\mu;\nu} - P_{\nu;\mu} = P_{\mu;\nu} - P_{\nu;\mu} + P_\alpha T_{\mu\nu}^\alpha \quad 15$$

$$G_{\mu\nu}(P) = F_{\mu\nu}(P) + P_\alpha T_{\mu\nu}^\alpha \quad 16$$

$$D^2 = -iq\alpha E(P) + q\Sigma H(P) - iq\frac{1}{2}\gamma^\mu\gamma^\nu P_\alpha T_{\mu\nu}^\alpha \quad 17$$

For gravity $G_{\mu\nu}(P)$ is the new gravitomagnetic tensor. $T_{\mu\nu}^\alpha$ is the torsion tensor [4].

$$D^3 = -iq\gamma^\mu\gamma^\nu\gamma^\delta G_{\mu\nu\delta} \text{ with } G_{\mu\nu\delta} = P_{\mu;\nu;\delta} - P_{\mu;\delta;\nu} \quad 18$$

$$G_{\mu\nu\delta}(P) = P_\alpha R_{\mu\nu\delta}^\alpha \text{ with } R_{\mu\nu\delta}^\alpha \text{ the Riemann-Christoffel tensor [5].}$$

$$D^3 = -iq\gamma^0\gamma^1\gamma^2 P_\alpha R_{012}^\alpha - iq\gamma^0\gamma^1\gamma^3 P_\alpha R_{013}^\alpha - iq\gamma^0\gamma^2\gamma^3 P_\alpha R_{023}^\alpha - iq\gamma^1\gamma^2\gamma^3 P_\alpha R_{123}^\alpha \quad 19$$

$$D^4 = -iq\gamma^\mu\gamma^\nu\gamma^\delta\gamma^\lambda G_{\mu\nu\delta\lambda} \text{ with } G_{\mu\nu\delta\lambda} = P_{\mu;\nu;\delta;\lambda} - P_{\mu;\nu;\lambda;\delta} \quad 20$$

$$\Gamma_{00}^1 = g_{00}Mr^{-2} \quad 26$$

$$\Gamma_{01}^0 = g_{00}^{-1}Mr^{-2} \quad 27$$

$$\Gamma_{11}^1 = -g_{00}^{-1}Mr^{-2} \quad 28$$

$$\Gamma_{12}^2 = \Gamma_{13}^3 = r^{-1} \quad 29$$

$$\Gamma_{22}^1 = -g_{00}r \quad 30$$

$$\Gamma_{23}^3 = \cot\theta \quad 31$$

$$\Gamma_{33}^1 = -g_{00}r\sin^2\theta \quad 32$$

$$\Gamma_{33}^1 = -g_{00}r\sin^2\theta \quad 33$$

$$\Gamma_{33}^2 = -\sin\theta\cos\theta \quad 34$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \times \mathbb{I} \quad 35$$

$$\gamma^0 = g_{00}^{-1/2} \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad 36$$

$$\gamma^1 = -g_{00}^{1/2} \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix} \quad 37$$

$$\gamma^2 = -r^{-1} \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} \quad 38$$

$$\gamma^3 = -r^{-1}\sin^{-1}\theta \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix} \quad 39$$

Gravitomagnetic tensor defined in Schwarzschild's Metric

We are using $x^0=t, x^1=r, x^2=\theta, x^3=\phi$ with $G=c=1$, this metric is defined by [6]

$$ds^2 = (1 - \frac{2M}{r})dt^2 - (1 - \frac{2M}{r})^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 \quad 23$$

$$g_{00} = (1 - \frac{2M}{r}), g_{11} = -(1 - \frac{2M}{r})^{-1}, g_{22} = -r^2, g_{33} = -r^2\sin^2\theta \quad 24$$

$$g^{00} = (1 - \frac{2M}{r})^{-1}, g^{11} = -(1 - \frac{2M}{r}), g^{22} = -r^{-2}, g^{33} = -r^{-2}\sin^{-2}\theta \quad 25$$

$$D^2 = im_\alpha E(P) - m\Sigma H(P) + im\frac{1}{2}\gamma^\mu\gamma^\nu P_\alpha T_{\mu\nu}^\alpha, \text{ from equations (16) and (17)} \quad 40$$

$$G_{12} = -H_3(P) = (-g_{00}^{1/2})(-r^{-1})(P_{1,2} - P_{2,1}) \quad 41$$

$$G_{13} = H_2(P) = (-g_{00}^{1/2})(-r^{-1}\sin^{-1}\theta)(P_{1,3} - P_{3,1}) \quad 42$$

$$G_{23} = -H_1(P) = (-r^{-1})(-r^{-1}\sin^{-1}\theta)(P_{2,3} - P_{3,2}) \quad 43$$

$$G_{03} = -E_3(P) = (g_{00}^{-1/2})(-r^{-1}\sin^{-1}\theta)(P_{0,3} - P_{3,0}) \quad 44$$

$$G_{02} = -E_2(P) = (g_{00}^{-1/2})(-r^{-1})(P_{0,2} - P_{2,0}) \quad 45$$

$$G_{01} = -E_1(P) = (g_{00}^{-1/2})(-g_{00}^{1/2})(P_{0,1} - P_{1,0}) \quad 46$$

$$T_{\mu\nu}^\alpha = 0 \text{ and } R_{012}^\alpha = R_{013}^\alpha = R_{023}^\alpha = R_{123}^\alpha = 0 \quad 47$$

Energy-momentum form is a 1-form [7]

$$\mathbf{p} = E dt - p_x dx - p_y dy - p_z dz \quad 48$$

$d\mathbf{p}$ is a 2-form

$$\mathbf{G} = d\mathbf{p} = E_x dt \wedge dx + E_y dt \wedge dy + E_z dt \wedge dz - B_x dy \wedge dz - B_y dz \wedge dx - B_z dx \wedge dy \quad 49$$

$$G_{32} = p_{z,y} - p_{y,z} \quad 50$$

$$G_{13} = p_{x,z} - p_{z,x} \quad 51$$

$$G_{21} = p_{y,x} - p_{x,y} \quad 52$$

Comparing equations (41-43) and (50-52) we can infer

$$P_\alpha = p_\alpha \quad 53$$

D^0 is related to the scalar 0-form m , D^1 is related to the Energy-momentum 1-form, D^2 is related to the Electromagnetic 2-form, D^3 is related to $\ast\mathbf{J}$ 3-form [8]

$$\begin{pmatrix} \ast J_{123} \\ \ast J_{023} \\ \ast J_{013} \\ \ast J_{012} \end{pmatrix} = \begin{pmatrix} -\rho \\ j_1 \\ -j_2 \\ j_3 \end{pmatrix} \quad 54$$

D^4 is related to \mathbf{L} 4-form [9]

$$\mathbf{L} = L_{0123} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \quad 55$$

$\gamma^p = -i\gamma^{14}$ is the projector matrix, historically γ^5 , but $\gamma^5 = \gamma^0 \gamma^2$

$$\gamma^p = -i\gamma^0\gamma^1\gamma^2\gamma^3 = (g_{00}^{-1/2})(-g_{00}^{1/2})(-r^{-1})(-r^{-1}\sin^{-1}) \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad 56$$

Torsion Tensor in a Rearranged Kerr's Metric

We are using $x^0=t, x^1=r, x^2=\theta, x^3=\phi$ M is the black hole's mass and a is the angular momentum per unit mass with $G = c = 1$. The invariance of the length of vectors under parallel transport means that the connection is compatible with the metric, it is a metric connection, the requirement of the preservation of the length by parallel transport may be stated as [10].

$$g_{\mu\nu;\sigma} = 0 \quad 57$$

$$g_{\mu\nu;\sigma} = g_{\mu\nu,\sigma} - g_{\alpha\nu}\Gamma_{\mu\sigma}^\alpha - g_{\mu\alpha}\Gamma_{\nu\sigma}^\alpha \quad 58$$

$$0 = g_{\mu\nu,\sigma} - g_{\nu\alpha}\Gamma_{\mu\sigma}^\alpha - t_{\alpha\nu}\Gamma_{\mu\sigma}^\alpha - g_{\mu\alpha}\Gamma_{\nu\sigma}^\alpha, \text{ with } t_{\mu\nu} = g_{\mu\nu} - g_{\nu\mu} \quad 59$$

$$0 = g_{\mu\nu,\sigma} - g_{\nu\alpha}\Gamma_{\mu\sigma}^\alpha - t_{\alpha\nu}\Gamma_{\mu\sigma}^\alpha - g_{\mu\alpha}\Gamma_{\nu\sigma}^\alpha \quad 60$$

$$g_{\mu\alpha}\Gamma_{\nu\sigma}^\alpha + g_{\nu\alpha}\Gamma_{\mu\sigma}^\alpha + t_{\alpha\nu}\Gamma_{\mu\sigma}^\alpha = g_{\mu\nu,\sigma} \quad 61$$

$$\Gamma_{\mu\nu\sigma} + \Gamma_{\nu\mu\sigma} + t_{\alpha\nu}g^{\lambda\alpha}\Gamma_{\lambda\mu\sigma} = g_{\mu\nu,\sigma} \quad 62$$

Solving these equations, we get the torsion applying its definition [11].

$$\Gamma_{\mu\nu\sigma} - \Gamma_{\mu\sigma\nu} = -T_{\mu\nu\sigma} \quad 63$$

Expanding the line element in powers of r^{-1} and examining the leading terms [12].

$$ds^2 = [1 - \frac{2M}{r} + O(r^{-2})]dt^2 + [\frac{4aM}{r}\sin^2\theta + O(r^{-2})]dtd\phi - [1 + O(r^{-1})][dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2] \quad 64$$

Rearranging the line elements

$$ds^2 = [1 - \frac{2M}{r} + O(r^{-2})]dt^2 + [O(r^{-2})]dtd\phi + [\frac{4aM}{r}\sin^2\theta]d\phi dt - [1 + O(r^{-1})][dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2] \quad 65$$

$$g_{00} = [1 - \frac{2M}{r} + O(r^{-2})] \quad 66$$

$$g_{03} = [O(r^{-2})] \quad 67$$

$$g_{11} = -[1 + O(r^{-1})] \quad 68$$

$$g_{22} = -[1 + O(r^{-1})]r^2 \quad 69$$

$$g_{33} = -[1 + O(r^{-1})]r^2 \sin^2 \theta \quad 70$$

$$g_{30} = \left[\frac{4aM}{r} \sin^2 \theta \right] \quad 71$$

$$t_{03} = [O(r^{-2})] - \left[\frac{4aM}{r} \sin^2 \theta \right] \quad 72$$

$$g^{00} = g_{33}(g_{00}g_{33} - g_{03}g_{30})^{-1} \quad 73$$

$$g^{03} = -g_{03}(g_{00}g_{33} - g_{03}g_{30})^{-1} \quad 74$$

$$g^{11} = g_{11}^{-1} \quad 75$$

$$g^{22} = g_{22}^{-1} \quad 76$$

$$g^{30} = -g_{30}(g_{00}g_{33} - g_{03}g_{30})^{-1} \quad 77$$

$$g^{33} = g_{00}(g_{00}g_{33} - g_{03}g_{30})^{-1} \quad 78$$

Generalizing Clifford algebra with $\delta_{\mu\nu}^{clif}$, if $\mu \neq \nu$ and $g_{\mu\nu} \neq 0$ then 1, else 0

$$\{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu} + g^{\nu\mu} - g^{\mu\nu} \delta_{\mu\nu}^{clif} - g^{\nu\mu} \delta_{\nu\mu}^{clif} \quad 79$$

$$\gamma^0 = [1 + O(r^{-1})]^{1/2} r \sin \theta (g_{00}g_{33} - g_{03}g_{30})^{-1/2} \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad 80$$

$$\gamma^1 = [1 + O(r^{-1})]^{-1/2} \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix} \quad 81$$

$$\gamma^2 = [1 + O(r^{-1})]^{-1/2} r^{-1} \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} \quad 82$$

$$\gamma^3 = \left[1 - \frac{2M}{r} + O(r^{-2}) \right]^{1/2} (g_{00}g_{33} - g_{03}g_{30})^{-1/2} \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix} \quad 83$$

$$D^2 = im\alpha E(P) - m\Sigma H(P) + im\frac{1}{2}\gamma^\mu \gamma^\nu P_\alpha T_{\mu\nu}^\alpha, \quad 84$$

from equations (16) and (17)

$$F_{12} = -H_3(P) = ([1 + O(r^{-1})]^{-1/2})([1 + O(r^{-1})]^{-1/2} r^{-1}) (P_{1,2} - P_{2,1}) \quad 85$$

$$F_{13} = H_2(P) = ([1 + O(r^{-1})]^{-1/2})([1 - \frac{2M}{r} + O(r^{-2})]^{1/2} (g_{00}g_{33} - g_{03}g_{30})^{-1/2})(P_{1,3} - P_{3,1}) \quad 86$$

$$F_{23} = -H_1(P) = ([1 + O(r^{-1})]^{-1/2} r^{-1})([1 - \frac{2M}{r} + O(r^{-2})]^{1/2} (g_{00}g_{33} - g_{03}g_{30})^{-1/2})(P_{2,3} - P_{3,2}) \quad 87$$

$$F_{03} = -E_3(P) = ([1 + O(r^{-1})]^{1/2} r \sin \theta (g_{00}g_{33} - g_{03}g_{30})^{-1/2}) ([1 - \frac{2M}{r} + O(r^{-2})]^{1/2} (g_{00}g_{33} - g_{03}g_{30})^{-1/2})(P_{0,3} - P_{3,0}) \quad 88$$

$$F_{02} = -E_2(P) = ([1 + O(r^{-1})]^{1/2} r \sin \theta (g_{00}g_{33} - g_{03}g_{30})^{-1/2}) ([1 + O(r^{-1})]^{-1/2} r^{-1})(P_{0,2} - P_{2,0}) \quad 89$$

$$F_{01} = -E_1(P) = ([1 + O(r^{-1})]^{1/2} r \sin \theta (g_{00}g_{33} - g_{03}g_{30})^{-1/2}) ([1 + O(r^{-1})]^{-1/2})(P_{0,1} - P_{1,0}) \quad 90$$

$\gamma^p = -i\gamma^{14}$ is the projector matrix, historically γ^5 , but $\gamma^5 = \gamma^0 \gamma^2$

$$\gamma^p = -i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = ([1 + O(r^{-1})]^{1/2} r \sin \theta (g_{00}g_{33} - g_{03}g_{30})^{-1/2}) ([1 + O(r^{-1})]^{-1/2}) ([1 + O(r^{-1})]^{-1/2} r^{-1}) ([1 - \frac{2M}{r} + O(r^{-2})]^{1/2} (g_{00}g_{33} - g_{03}g_{30})^{-1/2}) \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad 91$$

Conclusions

Multilevel operator $D^{mul}(g_{\mu\nu}, P_\nu, q)$ has been generalized for a curved space with a general four-potential P. For gravity $G_{\mu\nu}(P)$ is the new gravitomagnetic tensor and torsion tensor $T_{\mu\nu}^\alpha$ appears in its definition.

In a at space $G_{\mu\nu}(A) = F_{\mu\nu}(A)$, D^3 and D^4 operators vanish. In a curved space the curvature tensor $R_{\mu\nu\delta}^\alpha$ appears in levels 3 and 4.

The appearance of torsion tensor $T_{\mu\nu}^\alpha$ and curvature tensor $R_{\mu\nu\delta}^\alpha$ in multilevel operator $D^{mul}(g_{\mu\nu}, P_\nu, q)$ means that this operator is a fundamental operator in Quantum Field Theory.

$\gamma^0, \gamma^1, \gamma^2, \gamma^3$, have been calculated for Schwarzschild's metric, then $G_{\mu\nu}(P)$, the gravitomagnetic tensor has been obtained

Each D^n , where n is the number of γ matrices in the product of the algebra members, is related to an n-form.

The invariance of the length of vectors under parallel transport requires the vanishing of the metric tensor covariant derivative, a new term appears in equation (59) with $t_{\mu\nu} = g_{\mu\nu} - g_{\nu\mu}$ measuring the non-symmetric part of the metric tensor, solving these equations we get the torsion tensor.

Rearranging Kerr's metric we obtained $t_{03} = g_{03} - g_{30}$, the non-symmetric part of the metric tensor, gravitomagnetic tensor has also been calculated generalizing the Clifford algebra.

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