Appendix A Christoffel Symbol Calculations

For reference the metric is:

$$g_{\mu\nu} = \begin{pmatrix} -a^2 & & & \\ & a^2 & & \\ & & a^2r^2 & \\ & & & a^2r^2sin^2\theta \end{pmatrix}, \tag{6a}$$

and the equation for calculating Christoffell symbols is:

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}\left[\frac{\partial g_{\mu\beta}}{\partial x^{\nu}} + \frac{\partial g_{\nu\beta}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\beta}}\right].$$

The non-zero $\Gamma^{0}_{\mu\nu}$ values are:

$$\Gamma^{0}_{00} = \frac{1}{2}g^{00}\left[\frac{\partial g_{00}}{\partial x^{0}} + \frac{\partial g_{00}}{\partial x^{0}} - \frac{\partial g_{00}}{\partial x^{0}}\right] = \frac{1}{2}\left(\frac{-1}{a^2}\right)[-2a\dot{a}] = \frac{\dot{a}}{a},$$

$$\Gamma^{0}_{11} = \frac{1}{2}g^{00}\left[\frac{\partial g_{10}}{\partial x^{1}} + \frac{\partial g_{10}}{\partial x^{1}} - \frac{\partial g_{11}}{\partial x^{0}}\right] = \frac{1}{2}\left(\frac{-1}{a^2}\right)[-2a\dot{a}] = \frac{\dot{a}}{a},$$

$$\Gamma^{0}_{22} = \frac{1}{2}g^{00}\left[\frac{\partial g_{20}}{\partial x^{2}} + \frac{\partial g_{20}}{\partial x^{2}} - \frac{\partial g_{22}}{\partial x^{0}}\right] = \frac{1}{2}\left(\frac{-1}{a^2}\right)[-2a\dot{a}r^2] = \frac{\dot{a}}{a}r^2,$$

$$\Gamma^{0}_{33} = \frac{1}{2}g^{00}\left[\frac{\partial g_{30}}{\partial x^{3}} + \frac{\partial g_{30}}{\partial x^{3}} - \frac{\partial g_{33}}{\partial x^{0}}\right] = \frac{1}{2}\left(\frac{-1}{a^2}\right)[-2a\dot{a}r^2\sin^2\theta] = \frac{\dot{a}}{a}r^2\sin^2\theta.$$

The non-zero $\Gamma^{1}_{\mu\nu}$ values are:

$$\Gamma^{1}_{01} = \frac{1}{2}g^{11}\left[\frac{\partial g_{01}}{\partial x^{1}} + \frac{\partial g_{11}}{\partial x^{0}} - \frac{\partial g_{01}}{\partial x^{1}}\right] = \frac{1}{2}\left(\frac{1}{a^2}\right)[2a\dot{a}] = \frac{\dot{a}}{a},$$

$$\Gamma^{1}_{10} = \frac{\dot{a}}{a},$$

$$\Gamma^{1}_{22} = \frac{1}{2}g^{11}\left[\frac{\partial g_{21}}{\partial x^{2}} + \frac{\partial g_{21}}{\partial x^{2}} - \frac{\partial g_{22}}{\partial x^{1}}\right] = \frac{1}{2}\left(\frac{1}{a^2}\right)[-2a^2r] = -r,$$

$$\Gamma^{1}_{33} = \frac{1}{2}g^{11}\left[\frac{\partial g_{31}}{\partial x^{3}} + \frac{\partial g_{31}}{\partial x^{3}} - \frac{\partial g_{33}}{\partial x^{1}}\right] = \frac{1}{2}\left(\frac{1}{a^2}\right)[-2a^2r\sin^2\theta] = -r\sin^2\theta.$$

The non-zero $\Gamma^{2}_{\mu\nu}$ values are:

$$\Gamma^{2}_{02} = \frac{1}{2}g^{22}\left[\frac{\partial g_{02}}{\partial x^{2}} + \frac{\partial g_{22}}{\partial x^{0}} - \frac{\partial g_{02}}{\partial x^{2}}\right] = \frac{1}{2}\left(\frac{1}{a^2r^2}\right)[2a\dot{a}r^2] = \frac{\dot{a}}{a},$$

$$\Gamma^{2}_{20} = \frac{\dot{a}}{a},$$

$$\Gamma^{2}_{12} = \frac{1}{2}g^{22}\left[\frac{\partial g_{12}}{\partial x^{2}} + \frac{\partial g_{22}}{\partial x^{1}} - \frac{\partial g_{12}}{\partial x^{2}}\right] = \frac{1}{2}\left(\frac{1}{a^2r^2}\right)[2a^2r] = \frac{1}{r},$$

$$\Gamma^{2}_{21} = \frac{1}{r},$$

$$\Gamma^2_{33} = \frac{1}{2} g^{22}\left[\frac{\partial g_{32}}{\partial x^3} + \frac{\partial g_{32}}{\partial x^3} - \frac{\partial g_{33}}{\partial x^2}\right] = \frac{1}{2}\left(\frac{1}{a^2 r^2}\right)[-a^2 r^2 (2\sin\theta\cos\theta)] = -\sin\theta\cos\theta\,.$$

The non-zero $\Gamma^3_{\mu\nu}$ values are:

$$\Gamma^3_{03} = \frac{1}{2} g^{33}\left[\frac{\partial g_{03}}{\partial x^3} + \frac{\partial g_{33}}{\partial x^0} - \frac{\partial g_{03}}{\partial x^3}\right] = \frac{1}{2}\left(\frac{1}{a^2 r^2 \sin^2\theta}\right)[2a\dot{a}r^2\sin^2\theta] = \frac{\dot{a}}{a}\,,$$

$$\Gamma^3_{30} = \frac{\dot{a}}{a}\,,$$

$$\Gamma^3_{13} = \frac{1}{2} g^{33}\left[\frac{\partial g_{13}}{\partial x^3} + \frac{\partial g_{33}}{\partial x^1} - \frac{\partial g_{13}}{\partial x^3}\right] = \frac{1}{2}\left(\frac{1}{a^2 r^2 \sin^2\theta}\right)[2a^2 r\sin^2\theta] = \frac{1}{r}\,,$$

$$\Gamma^3_{31} = \frac{1}{r}\,,$$

$$\Gamma^3_{23} = \frac{1}{2} g^{33}\left[\frac{\partial g_{23}}{\partial x^3} + \frac{\partial g_{33}}{\partial x^2} - \frac{\partial g_{23}}{\partial x^3}\right] = \frac{1}{2}\left(\frac{1}{a^2 r^2 \sin^2\theta}\right)[a^2 r^2 (2\sin\theta\cos\theta)] = \cot\theta\,,$$

$$\Gamma^3_{32} = \cot\theta\,.$$

Appendix B Ricci Tensor and Scalar Calculations

For reference the non-zero Christoffell symbols are:

$$\Gamma^0_{00} = \frac{\dot{a}}{a},\ \Gamma^1_{01} = \Gamma^1_{10} = \frac{\dot{a}}{a},\ \Gamma^2_{02} = \Gamma^2_{20} = \frac{\dot{a}}{a}\,,$$

$$\Gamma^0_{11} = \frac{\dot{a}}{a},\ \Gamma^1_{22} = -r,\ \Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r}\,,$$

$$\Gamma^0_{22} = \frac{\dot{a}}{a} r^2,$$

$$\Gamma^0_{33} = \frac{\dot{a}}{a} r^2 \sin^2\theta,\ \Gamma^1_{33} = -r\sin^2\theta,\ \Gamma^2_{33} = -\sin\theta\cos\theta\,,$$

$$\Gamma^3_{03} = \Gamma^3_{30} = \frac{\dot{a}}{a},\ \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r},\ \Gamma^3_{23} = \Gamma^3_{32} = \cot\theta\,,$$

and the Riemann tensor equation is:

$$R^\alpha_{\beta\mu\nu} = \frac{\partial}{\partial x_\mu}\Gamma^\alpha_{\nu\beta} - \frac{\partial}{\partial x_\nu}\Gamma^\alpha_{\mu\beta} + \Gamma^\alpha_{\mu\gamma}\Gamma^\gamma_{\nu\beta} - \Gamma^\alpha_{\nu\gamma}\Gamma^\gamma_{\mu\beta}\,.$$

The R_{00} Ricci equation is

$$R_{00} = R^0_{000} + R^1_{010} + R^2_{020} + R^3_{030}\,,$$

and the pertinent Riemann tensors are:

$$R_{00} = 0,$$

$$R^1_{010} = 0 - \frac{\partial}{\partial x^0}\left(\frac{\dot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 - \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a}\,,$$

$$R^2_{020} = 0 - \frac{\partial}{\partial x^0}\left(\frac{\dot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 - \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a}\,,$$

$$R^3_{030} = 0 - \frac{\partial}{\partial x^0}\left(\frac{\dot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 - \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a}\,.$$

$$\rightarrow R_{00} = -3\left[\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2\right].$$

The R_{11} Ricci equation is

Citation: Proper GP (2023) A Solution to Einstein's Field Equations that Results in a Sign Change to the Analogous Friedmann Acceleration Equation. *OSP Journal of Physics and Astronomy* 4: JPA-4-151.

$$R_{11} = R_{101}^{0} + R_{111}^{1} + R_{121}^{2} + R_{131}^{3} ,$$

and the pertinent Riemann tensors are:

$$R_{101}^{0} = \frac{\partial}{\partial x^0}\left(\frac{\dot{a}}{a}\right) - 0 + \left(\frac{\dot{a}}{a}\right)^2 - \left(\frac{\dot{a}}{a}\right)^2 = \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2,$$

$$R_{111}^{1} = 0 ,$$

$$R_{121}^{2} = 0 - \left(\frac{-1}{r^2}\right) + \left(\frac{\dot{a}}{a}\right)^2 - \left(\frac{1}{r^2}\right) = \left(\frac{\dot{a}}{a}\right)^2,$$

$$R_{131}^{3} = 0 - \left(\frac{-1}{r^2}\right) + \left(\frac{\dot{a}}{a}\right)^2 - \left(\frac{1}{r^2}\right) = \left(\frac{\dot{a}}{a}\right)^2.$$

$$\rightarrow R_{11} = \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right].$$

The R_{22} Ricci equation is

$$R_{22} = R_{202}^{0} + R_{212}^{1} + R_{222}^{2} + R_{232}^{3} ,$$

and the pertinent Riemann tensors are:

$$R_{202}^{0} = \frac{\partial}{\partial x^0}\left(\frac{\dot{a}}{a}r^2\right) - 0 + \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{a}}{a}r^2\right) - \left(\frac{\dot{a}}{a}r^2\right)\left(\frac{\dot{a}}{a}\right) = \left[\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2\right]r^2,$$

$$R_{212}^{1} = \frac{\partial}{\partial x^1}(-r) - 0 + \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{a}}{a}r^2\right) - (-r)\left(\frac{1}{r}\right) = \left(\frac{\dot{a}}{a}\right)^2 r^2,$$

$$R_{222}^{2} = 0 ,$$

$$R_{232}^{3} = 0 - \frac{\partial}{\partial x^2}\cot\theta + \left[\left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{a}}{a}r^2\right) + \left(\frac{1}{r}\right)(-r)\right] - \cot^2\theta ,$$

$$= \csc^2\theta + \left(\frac{\dot{a}}{a}\right)^2 r^2 - 1 - \cot^2\theta = \left(\frac{\dot{a}}{a}\right)^2 r^2.$$

$$\rightarrow R_{22} = \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right]r^2.$$

The R_{33} Ricci equation is

$$R_{33} = R_{303}^{0} + R_{313}^{1} + R_{323}^{2} + R_{333}^{3} ,$$

and the pertinent Riemann tensors are:

$$R_{303}^{0} = \frac{\partial}{\partial x^0}\left(\frac{\dot{a}}{a}r^2\sin^2\theta\right) - 0 + \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{a}}{a}r^2\sin^2\theta\right) - \left(\frac{\dot{a}}{a}r^2\sin^2\theta\right)\left(\frac{\dot{a}}{a}\right) =$$

$$\left[\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2\right] r^2\sin^2\theta ,$$

$$R_{313}^{1} = \frac{\partial}{\partial x^1}(-r\ \sin^2\theta) - 0 + \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{a}}{a}r^2\sin^2\theta\right) - (-r\ \sin^2\theta)\left(\frac{1}{r}\right) = \left(\frac{\dot{a}}{a}\right)^2 r^2\sin^2\theta ,$$

$$R_{323}^{2} = \frac{\partial}{\partial x^2}(-\sin\theta\cos\theta) - 0 + \left[\left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{a}}{a}r^2\sin^2\theta\right) + \left(\frac{1}{r}\right)(-r\sin^2\theta)\right] - [(-\sin\theta\cos\theta)(\cot\theta)]$$

$$= (-\cos^2\theta + \sin^2\theta) + \left(\frac{\dot{a}}{a}\right)^2 r^2 \sin^2\theta - \sin^2\theta + \cos^2\theta = \left(\frac{\dot{a}}{a}\right)^2 r^2 \sin^2\theta,$$

$$R^3_{333} = 0.$$

$$\to R_{33} = \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] r^2 \sin^2\theta.$$

The Ricci scalar is:

$$R = R^0_0 + R^1_1 + R^2_2 + R^3_3.$$

$$R = -\frac{1}{a^2}\left[-3\frac{\ddot{a}}{a} + 3\left(\frac{\dot{a}}{a}\right)^2\right] + \frac{1}{a^2}\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] + \frac{1}{a^2 r^2}\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] r^2 + \frac{1}{a^2 r^2 \sin^2\theta}\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] r^2 \sin^2\theta$$

$$= 6\frac{\ddot{a}}{a^3}.$$

Citation: Proper GP (2023) A Solution to Einstein's Field Equations that Results in a Sign Change to the Analogous Friedmann Acceleration Equation. *OSP Journal of Physics and Astronomy* 4: JPA-4-151.